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A MODEL OF A MANPOWER TRAINING
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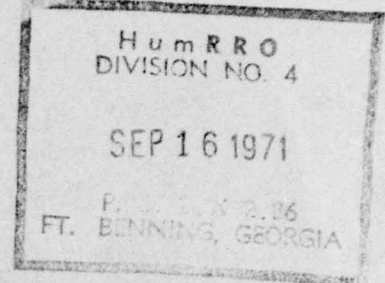# COMBAT TRAINING IN THE UNITED STATES ARMY

A THESIS

Presented to

The Faculty of the Division of Graduate

Studies and Research

By

John Edward Miller

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In Partial Fulfillment

of the Requirements for the Degree

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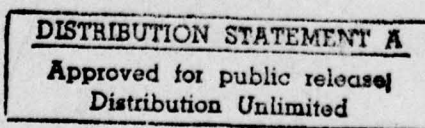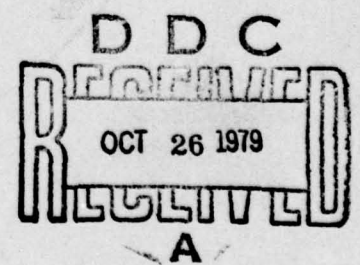
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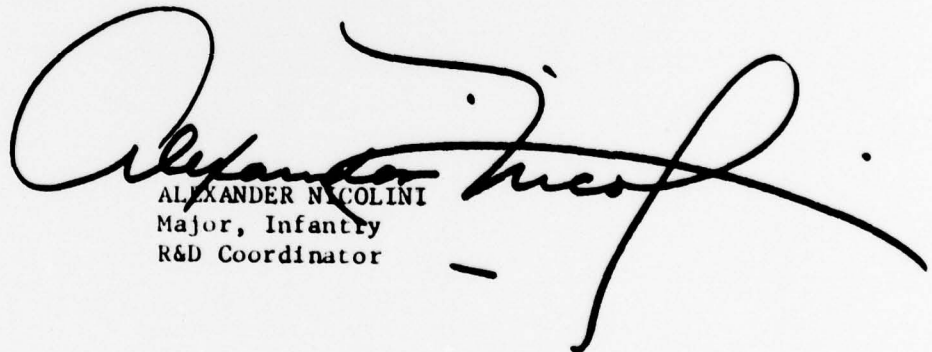
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Approved:

Norman R. Baker

Leslie G. Callahan Jr.

W P Rowe

Date approved by Chairman **May 24, 1971**

ACKNOWLEDGMENTS

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SUMMARY

Two performance criterion functions are developed and applied to a simulated manpower training system. The simulated system accepts inputs of varying quantity and quality. It contains multiple training events and may be operated such that each event has a probabilistic outcome. The cumulative probability of success for an event is a monotonically increasing function of time spent in that event and is independent of the order of the events. As the amount of time in an event is constrained, the maximum achievable probability of success within an event is lessened. The cost of operating the system is presented as a function of time, the probability of failure, the magnitude and quality of input, and the number of instructor hours per time unit. The performance criterion for the simulated training system is instructor hours per graduate.

Two different training procedures are compared using the system performance criterion of instructor hours per graduate. Within each procedure general relationships between input quantity and quality, output quantity, and cost of operation are developed, proven and demonstrated.

A rule for optimally sequencing the events in one of the training procedures is developed, proven and demonstrated. A general methodology for achieving the least cost, constrained output from an input of given size and quality is developed and demonstrated.

CHAPTER I

INTRODUCTION AND GENERAL PROBLEM STATEMENT

Introduction

This thesis presents the development of a quantitative performance criterion for a training system which accepts input of varying quality and which contains multiple training events. The system may be operated such that each event has a probabilistic outcome. The cumulative probability of success for an event is a monotonically increasing function of time spent in that event. A decision rule is developed for determining the most efficient sequence of the multiple events. Some men have the capability to complete one or several of the multiple events more quickly than others (12). It is shown that if men are allowed to progress at their own speed through the training system, the average training costs per man are reduced.

The results of this research have broad potential application. Although the solution methodology, criterion functions, lemmas, and theorem developed in this research were directed toward an analysis of a sequential training system, they are quite general in nature. The general nature of these findings enhances their potential for application to a large class of stochastic systems with similar characteristics. Two different training procedures are compared using a common criterion measure, cost per unit output. Within each procedure general relationships between input quality and quantity,

output quantity and the cost of operation are developed, proven and demonstrated.

Although this research resulted in a more "cost" effective training system design and a training system model, the effort was begun with the intention of describing how changes in the operating environment affected the Basic Combat Training (BCT) system. The relationships between the environment and the training system assumed in the construction of the performance criterion functions were that training performance was related to the quality of input and that performance and quality were both measurable. One example, BCT, fits these assumptions. Input from the environment in the form of men with individual aptitudes is related to the men's performance in the BCT training system. Performance is defined to be the ability to learn general military subjects. Both a man's aptitude for and performance in general military subjects have been quantitatively measured. These men, called "recruits" or "trainees" when input into BCT, vary in capability to perform well in BCT because of their individual aptitudes. The fluctuation of the general level of these aptitudes in input populations has changed over time and as will be shown, has had an impact on the BCT system. A study of the nature of this impact suggested that the present procedure should be redesigned.

General Problem Statement

Generally stated, the research problem is to develop a training procedure which will train men of highly diverse aptitudes in separate, aptitude-oriented training tracks. Each track should be designed to

emphasize those learning methods most appropriate to each aptitude group. The procedure should be designed so that each man may move through the training program as quickly as he is able. All men may not successfully master the training. Thus there must also be a method in the procedure for releasing training failures. To develop such a training procedure the following steps were taken:

1. The present system was examined and design weaknesses noted.

2. A system performance criterion was adopted (instructor hours per graduate).

3. A new training system design was developed.

4. Criterion functions were developed for the present system design and the proposed system design.

5. Both procedures were applied to a test training system under a variety of operating conditions. Their modeled behaviors were evaluated by the adopted system performance criterion.

6. The capabilities of the model of the proposed system were extended to include a rule for optimally arranging the event sequence in the training program.

The Example Environment

The United States Army requires that all enlisted men entering the active duty ranks complete BCT. BCT is normally an eight-week training program in which the novice soldier learns the basic skills necessary to survive in the field under training and combat conditions and how to live successfully in the Army community while in garrison. A trainee is expected to meet minimum performance standards in three major objective tests and in one subjective evaluation. The objective tests are:

1. Weapons qualification, normally accomplished in the fourth week of training.

2. Physical readiness, normally accomplished at or near the end of the eight-week training program.

3. Military subjects proficiency, normally accomplished at or near the end of the eight-week training program (3).

The subjective evaluation, "The Commander's Evaluation", is a written report of a man's overall performance in BCT as viewed by his training unit commander (1).

Researchers have discovered a definite relationship between an individual's performance on the Armed Forces Qualification Test (AFQT), his ability to learn, and his rate of learning general military subjects (13). The AFQT has also been found to be a reliable predictor of a man's performance in BCT as evaluated by his peers and cadre (6). No clear relationship between any measurement now taken as a man enters the Army and physical readiness performance has been established. Evidence has been found by researchers, however, which suggests that measurement devices do exist which are capable of predicting physical readiness performance over time. These measurement devices are simple in nature and appear to be suitable for administration at a U.S. Army Reception Station (11,15,19,20). No correlation between a man's ability with a rifle and any measurable aptitude has been found or suggested (8). No correlation between the commander's subjective evaluation and other measurable aptitudes has been found or suggested (8).

The environment in which Basic Combat Training is conducted has historically been a changing one, particularly in the last few years.

> . . . the problems surrounding the instruction of very low and very
> high academic abilities at the same time and in the same framework
> have become a matter of acute interest in the past few years. Tra-
> ditionally, military education systems, like civilian systems, have
> used a curriculum providing standard blocks of material to students
> of all aptitudes at the same time and pace. Attempts to individ-
> ualize treatment took the form of delaying the promotion of slow
> students and accelerating the progress of able students.
>
> In an earlier era, when civilian school groupings were moderately
> homogeneous after the seventh or eighth grade, this lockstep treat-
> ment of time and material did not present an insuperable problem.
> In recent decades, however, the extension of public schooling - and
> military training - to a vast range of students brought serious
> problems of communication and instruction from or related to the
> principle of lockstep instruction. Students handicapped by low
> ability, by difficulties in communicating, or by culturally influ-
> enced deficiencies have repeated work or have passed along from
> grade to grade without really learning tool subjects that are
> essential to learning in the typical occupational course or per-
> forming in a job. At the same time highly able students have been
> held back to the point of boredom and disinterest (18).

The Army draws its manpower resources from output of this single track
educational system and sets about training them in the same manner. As
Montague and Showel point out, logistical and administrative considera-
tions have in the past dictated that the Army continue using the single
track system (18).

> Since mid-1966, however, the introduction of a large number of
> men of lower aptitude from the draft and from enlistment has placed
> considerable strain upon the traditional instructional system and
> has reopened the question of how best to train men of such a wide
> range of ability as those now going through the (U.S. Army) train-
> ing centers (18).

Is dealing with a wide range of input capabilities really a significant
problem? In the most recent service history it has been. McFann in a
professional paper, "HumRRO Research on Project 100,000," (16) points
out:

. . . the Army trains a highly diverse population, varying over
time as the result of both the numbers of men needed and changes
in policies of enlistment and induction standards. The decision
to lower Armed Forces Qualification Test (AFQT) standards has
resulted in a large training load characterized by a wide spread
of individual ability and background ranging from elementary school
to college graduate level (16).

Fox, et al., further point out:

Since 1950 the Armed Forces Qualification Test (AFQT) has been
used by the Armed Services to determine an individual's eligibility
for military service. The AFQT, a written mental aptitude test,
is regarded as a general measure of trainability in military sub-
jects. A score falling at the tenth centile on the AFQT standardi-
zation distribution is the statutory minimum set by Congress for
acceptance into the military.

As the need for manpower has varied over time, the Armed Services
have adjusted their mental standards for enlistment and induction.
Following the Korean conflict the mental standards were gradually
raised, but in October 1966, under Project 100,000, the Department
of Defense announced its decision to lower mental standards for
induction to the statutory minimum.

The decision to implement Project 100,000 is resulting in large
numbers of marginal aptitude trainees appearing in the Army train-
ing program. Indications are that marginal aptitude trainees
(defined by AFQT centile scores ranging from 10 to 20) will con-
stitute about 25% of the input to the Army training system. This
increase in the number of marginal trainees will be likely to
increase the difficulty of the training job, requiring more effort
on the part of Army instructors to bring these people--with their
typical histories of difficulty and frustration in school activities--
up to minimum acceptable levels.

Anticipated training problems are not, however, limited to the
training of marginal aptitude personnel. It has been common
practice in military instruction to have students of all aptitude
levels enter a course together, use the same instructional materials,
progress at the same rate, and leave the course together. The
instructor, in order to keep attrition rates at a minimum, orients
his instruction to the slower trainees. This forces, on the entire
class, a slowed pace that may well have an adverse effect upon the
motivation and achievement of the higher aptitude trainees. Train-
ing will inevitably be diluted in an effort to reach the increasing
numbers of low aptitude people; consequently, a marked loss in
motivation and achievement by higher aptitude trainees may result
as they become even more bored and restless than evidenced in the
past. Thus, the cost to the Army of accepting large numbers of
men from the low end of the aptitude distribution may be twofold--
not only sheer difficulty in reaching those of marginal aptitude,
but also a negative impact upon higher level trainees.

It would seem axiomatic that the Army cannot achieve a standard, qualified training product by putting widely differing trainees through a standard training mold. Because trainees differ extensively in aptitude, education, and motivation, differential training may be necessary if they are to emerge with comparable skill levels at the end of training (13).

The present procedure used in BCT places all trainees, regardless of aptitude, together in groups of approximately 200. Each of these groups (training companies) is provided a training cadre, billets, and a training company commander. All trainees in a training company spend the same amount of time learning the same subjects (eight weeks). This procedure was applied to a sample training system for which a quantitative performance criterion function has been designed. An average quality input of 200 men was trained in the simulated system. The cost of training in terms of instructor hours per graduate was 13.7. As will be shown later, 13.7 instructor hours per graduate is less than optimally efficient.

It is apparent to those involved in studying BCT that the rates at which people with different aptitudes (variable input qualities) master many of the skills of BCT vary significantly (13). The design of the existing system does not easily permit the trainers to take advantage of the variable aptitudes of the trainees.

The training procedure proposed in this thesis, contrastingly, automatically differentiates between the learning rates of individuals. The proposed procedure allows multiple aptitude tracks, variable individual training completion times, and variable sequencing of the training events. The proposed procedure was applied to the same sample training system, under the same input conditions as used above

with the following results: The average cost in instructor hours per graduate was 5.70 for an increase in efficiency of eight less instructor hours per graduate. This represents a 58 percent increase in the training system's efficiency.

Changing the existing procedure for training recruits is not the only, or necessarily the best, method for improving the training system's ability to efficiently train men of different aptitude levels. This does not alter the fact that the efficiency of the present system is very strongly related to the rate of learning. But the rate at which people learn is related to many factors in addition to an AFQT score, some of which can be adjusted by the trainers. Examples of these factors might be the modes or methods of presenting instruction to different ability groups, the effect of attitude and morale on performance and the effects of incentive situations on performance. The Human Resources Research Office, Division 3, Recruit Training, continually studies ways to improve trainee learning rates. Their studies include the effects of different methods of teaching general military subjects on learning rates (13,21,23), the effects of attitude and morale on training performance, and the effects of BCT on trainee attitude and morale (22).

Baker has developed an application of a Charnes-Stedry model titled, "An Analytical Model of Worker Performance in Incentive Situations" (4). Baker's approach is to quantitatively predict worker goal statements based on five behavioral propositions when one of the following conditions exists:

1. An external goal is suggested.

2. A peer group goal is suggested.

3. Two external goals are suggested.

4. The worker is learning.

There are, no doubt, many other adjustable factors which can be influenced by the trainers to improve the performance of individuals in BCT.

The purpose of this thesis is not to prescribe the best teaching methods, the best attitude and morale boosters, or the best set of peer or organizationally suggested goals. It is however, to show that, given the present methods of instruction, morale, attitudes, and incentives, there does exist a system design for general military subjects training which uses fewer instructor hours per graduate than that now in use. Further, the model developed to predict the cost of running the system and its expected output allows one to vary the learning rates, the incremental cost in instructor hours, the input quantity and quality and finally, to constrain the output while still predicting costs and outputs. Optimal sequencing of the events in the system is possible as each variable above is discretely changed.

CHAPTER II

LITERATURE SEARCH

This literature search is directed toward two general areas of
investigation, the literature relevant to the present BCT system design
and individual performance in it, and the literature relevant to optimal
event sequences in stochastic systems.

The Present BCT System

Variable Input Capability

Basic Combat Training consists of training in four major subject
areas plus a two part proficiency test. These major subject areas and
test are:

1. Command Information and Indoctrination (17 hours training
    prescribed).

2. General Military Subjects (133 hours training prescribed).

3. Weapons Instruction (94 hours training prescribed).

4. Tactical Training (51 hours training prescribed).

5. Proficiency Testing (4 hours testing prescribed).

During mobilizations, for such as Viet Nam, 12 more hours of training
are added (1). A trainee can expect to spend 352 hours, 364 during
mobilizations, involved in formally scheduled activities during the
eight weeks of BCT. This usage of time accounts for approximately
50 percent of the time available for training during BCT (8 weeks x
6 days/week x 18 hours per day = 768 hours).

Over 30 experienced training company commanders, interviewed by this researcher during the period May 1967 to September 1967, at Fort Polk, Louisiana, indicated that an additional 24 to 26 hours per week were spent in unscheduled training. This was done in six of the eight weeks of training. These unit commanders felt that the additional, unscheduled training was necessary to raise their unit proficiency test score average to a level desired by the training center or major unit commander. One company commander commented that his unit spent only 10 to 12 hours in remedial training each week. At this one training center, then, a trainee could expect to spend 530 to 550 hours during a 768 hour period involved in controlled activities of other than his own choosing. This represents a 70 percent usage of all the hours available in the eight week program with no allowances for personal activities except that Sundays were left free.

HumRRO's Division 3 has also studied "Time Utilization during Basic Combat Training" (24). They subdivided a trainee's day as follows:

A - Scheduled Training.

B - Non-scheduled Activities Related to Training.

C - Non-scheduled Activities Not Directly Related to Training.

D - Sleep.

E - Personal Time.

F - Prescribed Non-Training Activities.

G - No Data.

HumRRO found that from Monday to Friday a trainee had 9 hours and 23 minutes of uncontrolled time (Categories D, E, and G) per day. On

Saturdays, D, E, and G amounted to 11 hours and 44 minutes (24). When weighed against 1152 total hours available for eight weeks, the HumRRO study results show 60 percent of a trainee's time is controlled. This study was performed at Fort Ord, California.

The purpose of reviewing the amount of controlled time in BCT is to acquaint the reader with how filled a trainee's schedule is already. This, combined with the fact that all the trainees in one training company operate on the same schedule (24), all beginning and ending activities together, are important background facts for the following discussions.

All men in BCT spend the same amount of time doing the same things. Some of these "things" involve learning general military subjects. If a man of high aptitude is capable of learning a subject in less than the time scheduled, and researchers have found that some men are (9,13), he must still spend the entire scheduled period learning that subject. Likewise, a man of low aptitude who cannot master the subject in the allocated time period must either be dropped from BCT or be trained beyond the allowable time by adding extra hours of remedial training or by recycling him in the training program. As the training week is so filled now, to add additional hours for extensive remedial training to the present training program is highly unlikely. Not only is the training week "saturated" now, one might also suspect the trainee is rather "saturated" with the training week.

What effect has variable trainee aptitudes had on BCT? BCT has become, as Fox, et al., point out,

> . . . highly standardized and pitched toward the level of the lower
> aptitude recruit. Not only is it elaborate and redundant but con-
> siderable effort is made - both in the formal training program and
> in individual, supplemental, remedial training - to ensure that
> almost all men meet graduation standards by passing the test. It
> resembles the public education system in the strong tendency for
> those who persevere in the system to graduate - witness the high
> percentage of low AFQT (Armed Forces Qualification Test) subjects
> who had completed high school (13).

Can these differences in aptitude be handled in the present BCT system?

Showel and Taylor comment on this point.

> For the most part, instruction in ATC's (Army Training Centers) is
> conducted as a single track system with minimum standards for grad-
> uation prescribed (21). Trainees enter together, receive the same
> program of instruction, and are programmed to graduate together (23).

Taylor goes on to say that a serious problem encountered with the single
track approach is in trying to decide at which level of aptitude to gear
the training.

> If it is at the low ability level, then the more capable are held
> back with the resulting boredom, poor attitude, and low efficiency
> of instruction. If instruction is generally geared to the upper
> level, the situation produces many who are failures (unduly high
> attrition rates) or many who are moved forward without mastering
> the material (23).

It is evident that variable input aptitudes are having an impact
on the present BCT system. Some doubt has been raised as to the present
system's ability to handle variable input qualities. The implication
seems to be that aptitude can be used to predict learning ability,
learning ability to predict individual performance and collective
individual performances to predict the operation of the BCT system.
If this is true, introducing a large number of either high or low
aptitude trainees into the system would be expected to cause a re-
gearing of the level of instruction to the detriment of the other
aptitude levels in the system. If the relationship between aptitude

and performance could be established, perhaps the relationship between BCT and its environment can be quantified.

Individual Aptitudes and Their Relationship to Performance

General Military Subjects. Bayroff, et al., conducted one of the earliest research efforts which attempted to relate scores on the present AFQT, a general aptitude measurement device, with BCT performance. They obtained AFQT scores from 498 men in their eighth week of BCT. These individual AFQT scores were correlated with a qualitative BCT performance measure (peer and cadre ratings). A coefficient of correlation of .44 was achieved. This was interpreted as a significant correlation (6).

In 1969, a study was conducted by Fox, et al., in which AFQT scores were correlated with objectively evaluated performance in general military subjects. Fox, Taylor, and Caylor (13) conclude the following in the report on their study of aptitude levels and skill acquisition in military training.

(1) Mental aptitude, as measured by the AFQT (Armed Forces Qualification Test), related consistently to a variety of important psychometric and operational criteria, including:
   (a) Performance on the Army's psychometric tests for classification and assignment.
   (b) Scholastic achievement as indicated by scores on reading and arithmetic tests, by school grade completed.
   (c) Army basic training performance as shown on a wide variety of tests of knowledge and skill in cognitive and motor subject areas, and a measure of leadership potential.
(2) Learning performance is directly related to aptitude levels. This relationship holds across a variety of training tasks which differ in complexity. This relationship is demonstrated by an array of response measures which show that:
   (a) In some tasks, aptitude groups differ only in rate of learning.
   (b) In some tasks, aptitude groups differ in rate of learning and in final level of performance.

     (c) In simple response tasks, aptitude groups differ in both
speed and accuracy of response.

     (d) The time required to train low aptitude recruits and high
aptitude recruits of comparable levels differs substan-
tially.

     (e) The learning performance of middle aptitude groups is more
similar to that of high aptitude groups than it is to low
aptitude groups.

     (f) Performance variability relates inversely to aptitude
level. Not all recruits labeled as being of low aptitude
are slow learners on all tasks; on each task a few show
performance typical of the middle and high aptitude groups.

     (g) The requirement for instructor guidance and prompting is
related inversely to aptitude level (13).

To illustrate the variable ability to learn of people with high,
average, and low AFQT scores (aptitudes), Fox, et al., structured a
training/testing program of seven events by which a test group was
trained. The results of this experiment are summarized graphically at
Appendix A. Why were these seven events chosen? Generally, they
represent the range of complexity and type learning activities en-
countered in BCT (13). The events are described below.

Event 1, "Simple and Choice Monitoring Tasks." This type of task
is typical of a variety of military tasks which require " . . .
visual surveillance or watch keeping activity. (e.g. switchboard
operators, fire (artillery) control personnel, sentries)" (13).

Event 2, "Rifle Assembly." This is a task specifically included
in BCT (13).

Event 3, "Rifle Disassembly." This is a task specifically in-
cluded in BCT (13).

Event 4, "Missile Preparation. . . . a fixed procedure task
which emphasizes learning a series of verbal responses." (e.g.
missile checkout procedures, engine trouble shooting, setting
fuses and preparing charges (demolitions), and checking out
radios (13).

Event 5, "Military Symbols" (13). A multiple discrimination task.

Event 6, "Phonetic Alphabet" (13). A multiple discrimination task.
BCT examples of the miltiple discrimination task group are, ". . .
learning hand and arm signals, . . . part names, . . . weapon
nomenclature, and color coding" (13).

Event 7, "Combat Plotting" (13). This task requires that principles
be learned and then applied. The concepts of range and bearing had
to be learned and applied. This task represents the highest level
of complexity. "Similar tasks in BCT are map reading and rifle
sight adjustment" (13).

In an attempt to generalize individual characteristics versus
performance, Cotham presented an approach for establishing a relation-
ship between individual characteristics and losses from a training
system. His analysis was not performed on a basic training model but
during ". . . three experiments using data on salesmen's characteristics
and performance histories collected in retail stores . . ." His ob-
jective was to, ". . . evaluate the potential of multiple discriminant
analysis used in conjunction with simple correlation analysis as a
technique for selecting candidates for sales positions in retail firms"
(12). In his report, Cotham demonstrates a methodology for correlating
selected individual characteristics with success in selling by using
historical data. The approach used by Cotham is generally the same
as that used by Fox, et al.

Clearly, then, aptitude as measured by the AFQT is related to
learning performance. This relationship exists in a variety of tasks
typical of BCT as demonstrated by Fox, et al. Further, Fox, et al.,
demonstrate that the rates at which people of various aptitudes learn
are related to their aptitudes. Thus the speed at which one learns a
skill to some final performance level as well as the height of the
final performance level are functions of aptitude. Fox, et al., also
point out that low aptitude trainees require longer to train in some
tasks. All of these relationships greatly increase the probability

of establishing a quantitative relationship between the variety of aptitudes in an input population and the BCT system.

Physical Readiness. Is there a relationship between individual aptitudes and physical readiness performance in BCT? There is no literature which specifically examines physical training in BCT. There are, however, some publications which indicate that such things as physical preconditions exist and further that these preconditions determine the rate of improvement during physical training and, ultimately, the level of performance.

Major Kenneth Cooper, M.D., has, ". . . evaluated, in one form or another, more than 5,000 subjects. These included officers and airmen, pilots and astronauts, athletes and non-athletes, the active and the inactive, the healthy and the unhealthy, and men and women - both in the field and in the laboratory" (11). His conclusion is that general fitness and cardiovascular efficiency are the same.

> To begin, I will give you a simple field test to perform that
> will establish your present physical condition . . . . The test
> will place you in one of several basic categories of fitness, each
> with its own graduated rate of progress . . . . If you have no
> serious ailments, you should be in good condition within 16 weeks
> at the most (11).

The evaluation or diagnostic test consists of running/walking as far as a man can in 12 minutes. Present condition is a function of distance over time.

The five categories of fitness as expressed by Cooper are:

> I. (Very Poor). Less than one mile on the 12-minute test,
> meaning your maximum oxygen consumption is less than 28 ml's/min.
> II. (Poor). Less than 1.25 miles and 34 ml's/min . . . .
> III. (Fair). Up to 1.5 miles and 42 ml's/min . . . .
> IV. (Good). More than 1.5 miles and 42 ml's/min . . . .
> V. (Excellent). Better than 1.75 miles and more than 52
> ml's/min . . . (11).

The physical training programs recommended by Cooper vary in initial intensity and rate of increase in intensity as a function of an individual's starting category. For example, a man in the Very Poor category would walk one mile in about 13:00 minutes for the first three weeks of a 16 week conditioning program. By the end of 16 weeks he could be running two miles in 17:00 minutes. A man whose starting category was Fair would walk one mile in about 12:45 minutes for the first week, walk and run a mile during the second and third weeks in ever decreasing times. This man could run two miles in 17:00 minutes by the end of the tenth week (11).

Cooper's interest in cardiovascular efficiency and O_2 uptake (aerobic power) are not unique. These subjects are given a great deal of attention by Astrand and Rodahl in their Textbook of Work Physiology (3). They also point out other factors that might affect physical performance.

> . . . the performance capacity (of an individual) is related to the maximal oxygen uptake in exercises with large muscle groups rigorously involved for 1 minute or longer. No one can attain top results in such exercises without a high degree of aerobic power. On the other hand, a high power does not guarantee a good performance, since technique and psychological factors may have a modifying influence in a positive or negative direction (3).

Physical fitness as measured through the Cooper test may not then be the only aptitude that deserves measurement when attempting to predict physical performance. Brace supports this point and specifically identifies some measurement techniques in the following comments:

. . . there is a substantial relationship between motor learning
of the sport-skill type and athletic ability, and between such
motor learning and physical fitness as measured by physical per-
formance level tests. The relationship with athletic ability,
however, is slightly closer than with physical fitness.

These findings substantiate those of previous studies in in-
dicating that learning of gross bodily motor skills of the sport-
skill type relates more closely to the qualities measured by tests
of running speed, jumping, and throwing than with motor ability
tests (the Brace Test), or with other standardized tests proposed
as measures of motor learning (7).

To return briefly to aerobic power, Astrand, et al., indicate
that there is a strong genetic connection to what one's maximal O_2
uptake is.

It is in any case quite obvious that the great maximal aerobic
power which is characteristic for the top athlete in endurance
largely depends on organic advantages which are endowed. Thus
a person with a maximal oxygen uptake of 45 ml/(kg)(min) cannot,
under any circumstances, no matter how much he trains, attain a
maximal oxygen uptake of 80 ml/(kg)(min) (3).

Astrand and Rodahl do, however, go on to indicate that by training, one

may approach his maximal O_2 uptake, lower his heart rate during vigorous

activity and generally improve the degree of efficiency with which his

body uses the O_2 taken in thereby improving his overall performance

capability for gross work (3).

No research was discovered which presented a relationship

between trainee attributes or aptitudes and performance in physical

readiness training. Research results were found which indicate that

a population can be subdivided into physical ability groups by means

of simple, easily administered tests (11,15,19,20). It is also known

that physical training activities must be milder initially and progress

slower for the initially less fit individual. If sufficient time is

available and if a man has no physiological defects he can achieve a

high degree of fitness regardless of his starting condition. The less fit initially, the longer the training program must be (11). These findings suggest that there are measurable individual abilities which are related to progress in physical fitness training. It has not been empirically established, however, that there is any relationship between the Physical Combat Proficiency Test (PCPT) performance and individual fitness. Thus no attempt is made by the author to include physical performance measures in the criterion functions for BCT.

Rifle Marksmanship. No study or research was found which describes or suggests a relationship between individual weapons training in BCT and any personal aptitude.

Summary. The only relationship then that has been clearly and quantitatively established between individual aptitudes and performance is that developed by Fox, et al., in the study of performance in general military subjects (13). This relationship will be used to develop a performance criterion for the BCT system, model the present system, redesign the present system, and model the redesigned system.

The Aptitude-Performance Relationship

The relationship between AFQT scores and the ability to learn general military subjects is a very important one to the establishment of a system performance criterion. Not only were Fox, et al., able to specify the learning rates by AFQT group for a variety of skills, they also collected data reflecting the number of instructor assists (prompts) by ability group required to assist the trainees in passing performance tests (See Appendix A) (13). These pieces of information allowed this researcher to develop a measure for the test system of the expected cost,

in terms of instructor hours, of training a man in a particular subject or event. This capability then allowed the calculation of the cost of training many men in different ability groups in six of the seven events used by Fox, et al., (13). The cumulative probabilities of success presented by Fox, et al., further allow the calculation of the expected output, given a particular input, for the test training system. These probabilities are presented graphically in Appendix A and in tabular form in Appendix D.

The combination of the ability to calculate the cumulative cost of training a given input in instructor hours and the expected output in numbers of trainees led quite logically to the selection of a system performance criterion of instructor hours per graduate. It was also decided at this point that six of the sample events used by Fox, et al., would constitute a test training system. (Only six of the seven events developed by Fox, et al., are used in the test system. Frequency of prompt data was not collected by Fox, et al., for one of the events, consequently that event could not be used in the test system) (13). This test or simulated system is used to demonstrate management applications of the model of the present BCT training procedure and the model of the redesigned BCT training procedure.

The Redesign of the Present BCT Training Procedure

Fox, et al., suggest that the relationship of performance with aptitude,

> . . . is a consistent and powerful one with important implications
> for the efficient conduct of training . . . . The efficient train-
> ing of men at all aptitude levels will depend on (a) the recognition
> of individual difference in aptitude, and (b) the design of the in-
> structional programs that are compatible with the individual differ-
> ences in learning rate and final performance capability (13).

Of primary interest to this researcher is, ". . . the design of instructional programs that are compatible with individual differences." The implications of the work by Fox, et al., are thought to be that a multiple track, variable training completion time procedure should be adopted for BCT.

In the 1965 HumRRO Technical Report 32, Cline, et al., presented an "Evaluation of Four-Week and Eight-Week Basic Training for Men of Various Intelligence Levels" (9). The conclusions, recommendations, and implications of this study are:

. . . (1) A four week basic training program for high-aptitude men has been demonstrated to be as effective as the current eight-week program in the areas of military information and certain performance tests, when a specific teaching aid (the Prevue-Review) is introduced. With respect to average score on rifle marksmanship and physical fitness, the high-aptitude four-week companies were for practical purposes as efficient as the regular eight-week companies.

(2) Trainees at all levels of aptitude learn as much military information in four weeks (when the Prevue-Review technique is used in their training) as is normally learned in eight weeks by men of comparable intelligence. On performance tests, men of middle and low aptitude do benefit by the full eight weeks of training, although the high-aptitude men apparently make only minor gains in the additional time.

(3) With respect to rifle marksmanship and physical fitness, the additional four weeks' training in the traditional course yields somewhat better performance at all levels of intelligence (although as noted above, in some cases these differences are so small as to have little practical significance).

b. These results imply that, with some changes of emphasis within the curriculum and the introduction of certain aids to learning, a basic training program of less than eight weeks might be feasible, particularly for higher-aptitude personnel. It would appear that new methods of instruction could be effectively employed, especially for general subject areas, and that greater emphasis on performance-type activities such as weapons familiarization and physical conditioning might be desirable.

c. With regard to the greater proportion of trainees, it has not been established that a shorter training period would be practicable in all subject areas. However, this study has given indications of the direction of change, in curriculum emphasis and instructional technique, which could be expected to contribute to this end.

d. The effectiveness of the results obtained in a shortened training program suggest that it should also be possible to turn out an even better-trained soldier in the eight-week program if certain modifications in curriculum and instructional techniques were exploited.

e. Consideration should be given to research on two alternative training programs:

(1) A short course which would train inductees to the present level of skill and knowledge (probably geared to men of high or middle aptitude). While the need for accelerated training would be greatest under conditions of full mobilization, the problems of maximum utilization of peace-time draftees might lead the Army to consider training revisions which would speed up integration of parts of the inductee population into the Army's working force, the TO&E units.

(2) A standard-length course which would afford better and more intensive training than does the current program. The findings strongly suggest that training as now conducted is geared for the lower-aptitude soldier and that the more able man is not making full use of his capacities.

Depending on future conditions, either or both programs might be chosen by the Army for operational purposes (9).

The conclusions and recommendations of Cline, et al., support the findings of Fox, et al.

The findings of both the Fox and the Cline studies suggest that the effectiveness of the present BCT training system might be improved if training, in length and manner of presentation, could be varied to best suit the aptitude of the man being trained. A major portion of the remainder of this thesis is dedicated to the development and analysis of a BCT training procedure which allows the trainee to proceed through the training program at a pace commensurate with his individual aptitude for learning general military subjects.

The Multiple Event Optimal Sequence Rule

A training procedure is designed and modeled which is thought to be a procedure of greater efficiency than that now in use. The model is adapted to a computer program and tested for different inputs and

event sequences. The efficiency of the new design is influenced by rearrangements in the sequence of the six events in the test system. The question arises as to which ordering of the events would provide the most efficient arrangement. An investigation of the literature is conducted.

Mitten in providing "An Analytical Solution to the Least Cost Testing Sequence Problem," develops an optimal sequencing rule for a series of n different tests, each with a probability of rejection R_i and a cost C_i.

That is, 1. For each test j, compute the ratio C_j/R_j; 2. Run the test with the smallest value for the above ratio first, the one with the second smallest ratio second, . . ., and the test with the largest ratio last (17).

Conway, et al., develop a shortest process time rule under the condition of weighted measures of performance. They order jobs optimally such that the job with the greatest flow time to weighted measure ratio is first in the sequencing. They argue and prove that ordering by this rule minimizes the job sequencing time (10).

Baker, in an application of Mitten's work, developed a rule for selecting the optimal search sequence for a "user" in search of a source to satisfy his need. Each source has a related cost of accessing it (C_h) and a probability that the user's need will be satisfied (P_h). Generally, the rule says that one must first form the set of ratios of the probability of failing to satisfy the user's need for each source to its corresponding cost of access (P_h/C_h). These ratios are then ordered from greatest to least ($P_h/C_h \geq P_{h+1}/C_{h+1}$). The sources should then be ordered in a corresponding order. Baker

demonstrates that this rule guarantees an optimal source accessing sequence (5).

Baker's optimal sequencing rule turns out to be the inverse of that developed by Mitten ($P_h/C_h \geq P_{h+1}/C_{h+1}$ versus $C_i/R_i < C_j/R_j$, $j > i$). The sequencing rule developed by Conway, _et al_., is analagous to those presented by Baker and Mitten. If one replaces cost by service time and relative or weighted importance by probability of success, the Conway, _et al_., rule very closely resembles that presented by Mitten. Although each of the above developments supports the conclusions of the other two, the development presented by Baker was that used by this researcher. Although none of the above rules was directly applicable to the proposed training system model, the development presented by Baker served as a guide to the eventual development of a decision rule uniquely applicable to the presented model.

CHAPTER III

THE PROPOSED PROCEDURE AND THE TEST BCT SYSTEM

The Proposed BCT Procedure

The proposed training system is a multi-ability-track, variable-completion-time one. The procedural rules are:

(1) Divide the input population in three ability or aptitude groups based on their performance on the AFQ Test. These groups are identified as M_1: high aptitude, M_2: average aptitude, and M_3: low aptitude.

(2) Begin training each ability group in a training program designed to emphasize those learning methods most appropriate for each group. Generally, this implies a higher instructor to student density and slower rates of learning for the less apt trainees.

(3) As quickly as a man reaches the criterion skill level in an event (or subject) move him to the next event in the training program.

(4) When a man has successfully completed all events in the training program, he is output from the training system.

(5) If a man is incapable of satisfactorily mastering an event after a reasonable number of attempts, he is dropped from the training system as a failure. He does not move from one track to the next. The three training tracks are operated independently.

The following assumptions are made:

(1) Each training event consists of a set learning period followed by a test. A man completes a "trial" when he has received

the instruction in the learning period and has been tested. If he fails the test, a man repeats the entire trial.

(2) There is no dependency between events. That is, the probability of success for an event is independent of the event's order in the training sequence.

(3) To reach criterion level performance for a given event, a man need only pass one trial test. He then moves to the next event in the sequence.

(4) The term "a reasonable number of trials" mentioned in the paragraph above is construed to mean, based on the research done by Fox, Taylor, and Caylor (13), that further trials would be highly unlikely to produce success on the part of the tested trainee.

(5) For purposes of comparing the present and the proposed procedures, no trainee is lost under either procedure from the test system for administrative reasons. Such losses would occur in an equally likely number under either procedure as input populations are identical for the test system regardless of the operating procedure used. Examples of administrative losses are: punitive discharges, discharges for hardship reasons and desertions.

(6) The administrative, non-training costs of running the BCT system under either procedure are equal. This assumption is not at all based on fact or reasonable conjecture. The third procedural rule generates a highly likely source of increased administrative costs. Imagine 200 trainees moving at 200 different paces through a training program. As each trainee finishes event one, he is transported 15 miles to participate in event two. Compare that with the

present method of all 200 trainees simultaneously completing event one,
being transported simultaneously to event two and beginning event two
together. The administrative costs of the procedure proposed in this
example would be significantly greater than the cost of the present
method. A closer investigation of the proposed procedure, however,
reveals that men will not be moving through the training program at
200 different paces. The average expected output from each trial,
given an average quality input, is nine men in the high aptitude track,
14 men in the average aptitude track, and two men in the low aptitude
track. Thus trainees can be expected to complete events in small
groups. These small groups could be transported together in a common
carrier to the next event as in the present procedure. Consider, how-
ever, an arrangement of the training events similar to exhibits or
booths in a county fair. A trainee group, operating by the proposed
procedure, completes event one then moves down the midway to event two.
The quicker they proceed from events one to two, the quicker they are
likely to finish BCT. If a man finishes early perhaps he could then be
sent home on leave or take more advanced training. The administrative
costs of this last example might be quite low as the trainee is left
to administer himself. Before deciding conclusively, however, which
procedure is more cost efficient, some administrative methods for
handling the trainees under the proposed procedure must be developed
and quantitatively compared with the existing methods.

The Test System

The mock BCT system used to test the present and the proposed
training procedures is that presented in Chapter 2. Six of the seven

events used by Fox, et al., comprise the six event test general military subjects training system. The six adopted events are:

1. Rifle Assembly

2. Rifle Disassembly

3. Missile Preparation

4. Military Symbols

5. Phonetic Alphabet

6. Combat Plotting

As previously stated, learning rates for each event for the high, average, and low aptitude groups have been established. These learning rates were used to calculate the cumulative probability of success for each trial in each event (See Appendices A and D). The frequency of prompts by instructors was also recorded by Fox, et al., for each aptitude group in each event. These prompt frequency data were used in the calculation of the cost coefficients for each event (See Appendix E). Fox, et al., held the modes of instruction constant within a given event. The modes generally favored the less able trainee. The gearing of instructional modes to the low aptitude trainee is typical of the BCT system (13, 16). The format used by Fox, et al., within a training event consisted of a repetitive cycle of instruction-testing (a trial) with the emphasis on making each cycle as similar as possible (13).

Summary

The development of a new BCT training procedure and the adoption of a simulated test system representative of general military subjects

learning in BCT provide the elements necessary to compare the present and proposed training procedure. To make a simulated quantitative comparison of the two procedures using the test system data, it was necessary to develop mathematical representations of each procedure. This development is the subject of the next chapter.

CHAPTER IV

MODELS AND METHODOLOGY

The Development of the Criterion Functions
For the Present and Proposed Procedures

The relationships which determine the expected output, the cost
of operation and the efficiency of the simulated training system operated
by the present and the proposed procedures are quite general in nature.
The general nature of these criterion functions and the methodologies
for constraining input and output and optimally ordering the training
event sequence enhance their potential for broad application.

The Present Procedure

Men are trained in a series of subjects (events) and then given
a performance test at the end of the entire training program. The
number of men being trained remains constant throughout the training
program. Men are lost from the system only at the final tests.
Graphically:

Event   1   2 . . .   k   . . .   n   Final Tests

Input M_I ──▶[1]─M_I─▶[2]─M_I─▶[k]─M_I─▶[n]─M_I─│ $PM_I = M_o$
                                                                  │ Output

Event C_1 + C_2 + ... + C_k + ... + C_n
Cost
                                                   Losses $= (1-P) M_I$
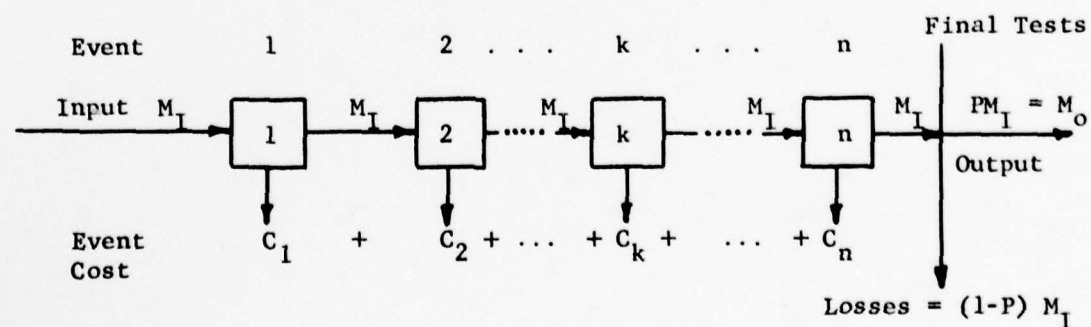
Figure 1. The Present Procedure.

Total cost $= M_I \sum\limits_{k=1}^{n} C_k = C$. System efficiency $= EI = \dfrac{C}{M_O}$. Symbolically, the criterion measures for the present procedure may be represented as follows:

$$\text{Total cost, } C = M_I \sum_{k=1}^{n} C_k \quad \text{as above.} \tag{1}$$

$$\text{Expected output, } M_O = M_I P$$

$$\text{System Efficiency Index, } EI = \frac{C}{M_O} = \frac{M_I \sum\limits_{k=1}^{n} C_k}{M_I P} \quad , \text{ therefore} \tag{2}$$

$$EI = \frac{\sum\limits_{k=1}^{n} C_k}{P} \quad . \tag{3}$$

In the above equations, the unidentified variables and subscripts are:

k = event index, $k = 1, 2, \ldots, n$.

i = aptitude track index, $i = 1, 2, \ldots, s$.

M_i = input in the ith aptitude track.

M_I = Total input $= \sum\limits_{i=1}^{s} M_i$.

C_k = Cost of training one man in event k.

P = Probability of success of passing all the final tests.

The Proposed Procedure

Men are trained and tested a trial at a time within an event. There is a probability of success associated with each trial level within each event within each aptitude group, P_{ijk}, where,

i = aptitude level index, i = 1, 2, . . ., s.

j = trial level, that is the number of trials completed,
    j = 1, 2, . . ., a.

k = event index, k = 1, 2, . . ., n.

As j, the number of trials completed within an event increases, the value of P_{ijk} increases monotonically. The maximum probability of success possible for any event within an aptitude track is a function of the highest number of trials to be run in that event. There are multiple trials within an event, multiple events within an aptitude track and multiple aptitude tracks within the system. Training is conducted within independently operating aptitude tracks, which assumes that men can be categorized by aptitude group. Men pass after each trial, but fail only after the last trial within an event is completed. The most significant distinction between the proposed procedure and the present one is that in the proposed training procedure men who pass a trial test within any event are finished training in that event. They proceed directly to the next event regardless of whether their fellow trainees have passed the same trial test, whereas in the present procedure the men all move to each event from the preceding one in a group. A schematic representation of a single aptitude track for the proposed procedure is presented in Figure 2.

The criterion measure for this procedure contains the following variables and their subscripts. The subscripts are:

i = the aptitude track (i = 1, 2, . . ., s).

j = cumulative number of trials (j = 1, 2, . . ., a_{ik}).

k = the event identifier (k = 1, 2, . . ., n).

Event

M_i = Input

1

$M_i P_{i11}$ Input

2

$M_i P_{ia_{i1}} P_{i12}$

. . .

s

M_{io} = Output

| Trial 1 | Output |

$M_i P_{i21}$ Output

Trial 2

$M_{io} = M_i \prod_{k=1}^{s} P_{ia_{is}}$

Trial 1

Costs

Trial Failures
$M_i(1-P_{i11})$

$M_i(1-P_{i21})$

Costs

Trial Failures
$M_i P_{ia_{i1}}(1-P_{i12})$

Costs

Trial Failures
$s-1$
$M_i \prod_{k=1}^{s-1} P_{ia_{ik}}(1-P_{i1s})$

$M_i(1-P_{i,a_{i1}-1,1})$

$M_i P_{ia_{i1}}$ Output

Trial a_{i1}

$M_i \prod_{k=1}^{2} P_{ia_{ik}}$

Trial a_{i2}

$s-1$
$M_i \prod_{k=1}^{s-1} P_{ia_{ik}}$

Trial a_{is}

$M_i(1-P_{ia_{i1}})$

$M_i P_{ia_{i1}}(1-P_{ia_{i2}2})$

$M_i \prod_{k=1}^{s-1} P_{ia_{ik}}(1-P_{ia_{is}})$

Costs

$C_{i1}M_i$

+

$C_{i1}M_i(1-P_{i11})$

+

$C_{i1}M_i(1-P_{i,a_{i1}-1,1})$

a_{i1}
$C_{i1}M_i \sum_{j=0} (1-P_{ij1})$

+

Event Failures

a_{i2}
$+ C_{i2}M_i P_{ia_{i1}1} \sum_{y=0} (1-P_{ijy=0})$

+

Event Failures

$s-1$ a_{is}
$+ C_{is}M_i \prod_{k=1} P_{ia_{ik}} \sum_{isj=0} (1-P_{ijs})$ = Total Costs

+ Event Failures = Total Losses

Figure 2. The Proposed Procedure.

The variables are:

P_{ijk} = the cumulative probability of success for a man in the ith aptitude group after j trials in event k of the test system.

M_i = number of men input into the ith aptitude track.

a_{ik} = the maximum number of trials allowed for men of aptitude track i in the kth event.

C_{ik} = cost (instructor hours per man per trial) for training a man of aptitude track i in the kth event for one trial.

An incremental training cost is incurred each time a man is trained and tested (completes a trial). This cost continues to accumulate until the man reaches the established performance criterion or until he is dropped from the program as a failure at the end of an event. These incremental costs vary between aptitude groups and events but remain constant between trials within an event and aptitude track. Given an aptitude level, i, in any event, k, the expected cost of training personnel in the jth trial is equal to the cost per trial per man times the number of men being trained in that trial. The cost per man per trial for aptitude level i in the kth event is known, C_{ik}. The expected number of men being trained in the jth trial is equal to the expected number of men starting that event minus the expected number of men who have reached the criterion in the preceding j-1 trials. That is $M_{ijk} = (M_{ik} - P_{i,j-1,k} M_{ik})$. The number of men, M_{ik}, input from event k-1 for aptitude level i to event k is equal to the expected output from the preceding k-1 events. That is:

$$M_{ik} = M_i \prod_{n=1}^{k-1} P_{ia_{ik}n}.$$

The cost of a single event, k, then is the sum of all the trial costs in that event for all s aptitude groups.

$$C_k = \sum_{i=1}^{s} M_i \, C_{ik} \sum_{\ell=0}^{k-1} P_{ia_{i\ell}\ell} \sum_{j=0}^{a_{ik}} (1 - P_{ijk}) \tag{4}$$

The cost of all n events in the test system may be shown as

$$C = \sum_{i=1}^{s} \sum_{k=1}^{n} M_i \, C_{ik} \prod_{\ell=0}^{k-1} P_{ia_{i\ell}\ell} \sum_{j=0}^{a_{ik}} (1 - P_{ijk}) \tag{5}$$

subject to j, an integer, $M_i \geq 0$, and $0 \leq P_{ijk} \leq 1$.

The expected output of the system is the sum for all aptitude tracks of the product of the terminal cumulative probabilities of success for all the events in a given aptitude track times the original input of the given track. That is:

$$M_o = \sum_{i=1}^{s} M_i \prod_{k=1}^{n} P_{ijk} \quad \text{where } j = a_{ik}. \tag{6}$$

To operate the new system under the constraint of a minimal feasible total output without regard to quality of output, this additional constraint must be added:

$$\sum_{i=1}^{s} \prod_{\ell=1}^{n} [P_{ia_{i\ell}\ell} \, M_i] \geq M_o^*, \text{ the desired output.} \tag{7}$$

If a particular quality mix of output is desired such constraints as follows may be added:

$$\prod_{k=1}^{n} P_{ra_{rk}k} M_r \geq M_r^* ,$$ (8)

the desired output for the rth aptitude level. The efficiency index of the proposed system is

$$\frac{C}{M_o} = \frac{\text{Total Cost}}{\text{Expected Output}} .$$ (9)

EI = average cost per graduate for all aptitude tracks. The EI for any given aptitude track, $i = r$, may be expressed as

$$EI_r = M_r \sum_{k=1}^{n} C_{rk} \sum_{\ell=0}^{k-1} P_{ra_{r\ell}\ell} \sum_{j=0}^{a_{rk}} (1 - P_{ijk})$$ (10)

EI_r, then, is independent of input within any given aptitude track and EI_r = average cost per graduate in the rth aptitude track. The total average cost of training a given output may be expressed as

$$EI = \frac{\sum_{i=1}^{s} EI_i M_i \prod_{k=1}^{n} P_{ia_{ik}k}}{\sum_{i=1}^{s} M_i P_{ia_{ik}k}}$$ (11)

where $M_i \prod\limits_{k=1}^{n} P_{ia_{ik}k} = M_i^*$.

An Analysis of the Present Procedure

The efficiency of the system operated by the present procedure as evaluated by equation (3) appears to be independent of the total input, M_I. A closer examination reveals that is not true, however. The event cost coefficients, C_k, are a function of the input by aptitude level and the event cost by aptitude level:

$$C_k = \frac{\sum\limits_{i=1}^{s} C_{ik} M_i}{\sum\limits_{i=1}^{s} M_i} .$$

P is a known constant. Then to improve the efficiency, Min EI, for the present system, $\sum\limits_{k=1}^{n} C_k$ must be minimized in equation (3).

Lemma 1:

(i) If $0 \le C_{ik} < 1$, for all i, k, then any integer reduction in $M_I = \sum\limits_{i=1}^{s} M_i$ causes an increase in C_k.

(ii) If $C_{ik} > 1$, for all i, k, then an integer reduction in $M_I = \sum\limits_{i=1}^{s} M_i$ causes a decrease in C_k.

(iii) If $C_{ik} = 1$, for all i, k, then a reduction in input has no effect on C_k.

Proof:

(i) Given,

$$C_k = \frac{C_{1k} M_1 + C_{2k} M_2 + \ldots + C_{rk} M_r + \ldots + C_{sk} M_s}{M_1 + M_2 + \ldots + M_r + \ldots + M_s} \qquad (12)$$

and $C_{ik} < 1$, for all i. Suppose: M_r is reduced by ΔM_r.

$$C_k' = \frac{C_{1k}M_1 + C_{2k}M_2 + \ldots + C_{rk}(M_r - \Delta M_r) + \ldots + C_{sk}M_s}{M_1 + M_2 + \ldots + M_r - \Delta M_r + \ldots + M_s} \quad (13)$$

$$C_k' = \frac{C_{1k}M_1 + C_{2k}M_2 + \ldots + C_{rk}M_r + \ldots + C_{sk}M_s - C_{rk}\Delta M_r}{M_1 + M_2 + \ldots + M_r + \ldots + M_s - \Delta M_r} \quad (14)$$

$$C_k' = \frac{\sum_{i=1}^{s} C_{1k}M_i - C_{rk}\Delta M_r}{\sum_{i=1}^{s} M_i - \Delta M_r} \quad \text{where } C_{rk} < 1. \quad (15)$$

Therefore $C_k' > C_k$.

(ii) From equation (15), if $C_{rk}' > 1$

$$C_k' = \frac{C_{ik}M_i - C_{rk}' \Delta M_r}{\sum_{i=1}^{s} M_i - \Delta M_r}$$

and, therefore, $C_k' > C_k$.

(iii) From equations (14) and (12),

$$C_k' = \frac{C_{ik}M_i - (C_{rk} = 1)\ \Delta M_r}{M_i - \Delta M_r} = \frac{M_i - \Delta M_r}{M_i - \Delta M_r} = 1, \tag{16}$$

where $M_i > \Delta M_r$, and

$$C_k = \frac{\sum\limits_{i=1}^{s} C_{ik}M_i}{\sum\limits_{i=1}^{s} M_i} = \frac{\sum\limits_{i} M_i}{\sum\limits_{i} M_i} = 1 \ . \tag{12}$$

Therefore $C_k' = C_k$.

Lemma 2. Let ΔM_o be a desired reduction in expected output and this will be attained by reducing input in one track. Then, in order to achieve the minimum increase or maximum decrease in C_k', reduce input in the highest cost aptitude track.

Proof. Suppose ΔM_o can be realized by reducing input to any one track.

$$C_k = \frac{\sum\limits_{i=1}^{s} C_{ik}M_i}{\sum\limits_{i=1}^{s} M_i} = \frac{C_{1k}M_1 + C_{2k}M_2 + \ldots + C_{sk}M_s}{M_1 + \ldots + M_s} \ . \tag{12}$$

Suppose input is reduced in track s, that is, M_s is changed to $(M_s - \Delta M_s)$. Then

$$C'_k = \frac{\sum_{i=1}^{s} C_{ik}M_i - C_{sk}(\Delta M_s)}{\sum_{i=1}^{s} M_i - \Delta M_s} \quad . \tag{16}$$

(i) Suppose all $C_{ik} < 1$. By Lemma 1 (i), $C'_k > C_k$. Since the reduction in the denominator depends only on ΔM_s, the amount of reduction, the change in the denominator is independent of the aptitude level. Thus, to minimize $(C'_k - C_k)$, C_{sk} must be as large as possible, that is as close to one as possible.

(ii) Suppose all $C_{ik} > 1$. By Lemma 1 (ii), $C_k > C'_k$. Since the reduction in the denominator depends only on ΔM_s, the amount of reduction, the change in the denominator is independent of the aptitude level. Thus to maximize $(C_k - C'_k) > 0$, C_{sk} must be as large as possible.

(iii) Suppose all $C_{ik} = 1$. By Lemma 1 (iii), $C_k = C'_k$. Thus changing the denominator by ΔM_s, the amount of the reduction, changes the numerator by ΔM_s also. Consequently $C_k = C'_k$ for all $\Delta M_s < M_s$.

It follows by the same reasoning that if ΔM_o cannot be achieved by continued input reductions in the highest cost track, continued reductions in input must be made in the next highest cost track.

Lemma 3. If the cost coefficients can be ordered such that $C_{sk} \geq C_{s-1k} \geq C_{s-2k} \cdots \geq C_{2k} \geq C_{1k}$, and the ordering holds for all k, then if reductions are desired, the minimum cost will result if the reductions are always made in the highest cost aptitude track.

Proof. Given

$$C = \sum_{k=1}^{n} C_k = \frac{\sum_{i=1}^{n} M_i \sum_{k=1}^{s} C_{ik}}{\sum_{i=1}^{n} M_i} \text{ and } C_{sk} \geq C_{s-1k} \cdots \geq C_{1k}$$

then it follows that

$$\sum_{k=1}^{n} C_{1k} \leq \sum_{k=1}^{n} C_{2k} \cdots \leq \sum_{k=1}^{n} C_{rk} \cdots \leq \sum_{k=1}^{n} C_{sk} .$$

Suppose we wish to make an input reduction of ΔM in some aptitude track. The change would result in a reduction in cost as follows.

$$C' = \sum_{k=1}^{n} C_k = \frac{M_1 \sum_{k=1}^{n} C_{1k} + \cdots + (M_r - \Delta M) \sum_{k=1}^{n} C_{rk} + \cdots + M_s \sum_{k=1}^{n} C_{sk}}{\sum_{i=1}^{s} M_i - \Delta M} .$$

(17)

(i) If $\sum_{k=1}^{n} C_{rk} < 1$, then it follows by Lemma 2 (i), $C < C'$. Choosing the track in which $\sum_{k=1}^{n} C_{rk}$ is as large as possible, namely $\sum_{k=1}^{n} C_{sk}$, minimizes the increase in cost, min $(C' - C)$.

(ii) If $\sum_{k=1}^{n} C_{rk} > 1$, then it follows by Lemma 2 (ii), $C' < C$. Choosing the track in which $\sum_{k=1}^{n} C_{rk}$ is as large as possible, namely $\sum_{k=1}^{n} C_{sk}$, maximizes the decrease in cost, max $(C - C')$.

(iii) Likewise by Lemma 2 (iii) when $\sum_{k=1}^{n} C_{rk} = 1$, $C' = C$ for any change in M_i.

An Analysis of the Proposed Procedure

The nature of the proposed procedure gives rise to some intriguing and very general conclusions.

Optimal Event Order

Theorem 1: If the events in any given aptitude track are ordered by the following procedure, the cost of training any given input is minimized.

(i) For each event k, compute the ratio

$$\frac{1 - P_{a_k k}}{C_k \sum_{j=0}^{a_{ik}} (1 - P_{jk})} \tag{18}$$

(ii) Place the event with the largest value for the above ratio first, the one with the second largest ratio value second, ..., and the event with the smallest ratio last.

Proof: Let C_i be the cost of any sequence for the ith aptitude track. The cost of operating all aptitude tracks, $C_s = \sum_{i=1}^{s} C_i$. But each aptitude track operates independently (Rule 5, Chapter 3, page 27). If Min $C_i = C_i^*$, i = 1, 2, ..., s, then it follows that

$$C_s^* = \text{Min } C_s = \sum_{i=1}^{s} C_i^* . \tag{19}$$

Therefore, if Min C_i^* can be found for each i, then C_s^* may be calculated directly. The following development shows how C_i^* may be found. (Note:

C_i^* is replaced by C^* for notational ease. That is, a particular aptitude track, i, has been selected. Likewise the subscript i and $\overset{s}{\underset{i=1}{\Sigma}}$ notation are deleted from equation (4) in the following discussion). Suppose one sequence called s has a minimum cost of operation, C_s^*, and another sequence, s', exists which is feasible and has a cost C_s, not a minimum. Further suppose that the only difference between these two sequences is that the h and h + 1 events are reversed in the feasible, nonoptimal sequence. It may be concluded that

$$C_s^* \leq C_{s'}, \text{ or } C_s^* - C_{s'} \leq 0 . \tag{20}$$

Substituting equation (1) into equation (7) and expanding yields:

$$M[C_1 P_o \overset{a_1}{\underset{j=0}{\Sigma}} (1 - P_{j1}) + \ldots + C_h \overset{h-1}{\underset{n=1}{\pi}} P_{a_n n} \overset{a_h}{\underset{j=0}{\Sigma}} (1 - P_{jh}) \tag{21}$$

$$\ldots + C_{h+1} \overset{h-1}{\underset{n=1}{\pi}} P_{a_n n} P_{a_h h} \overset{a_{h+1}}{\underset{j=0}{\Sigma}} (1 - P_{j,h+1}) + \ldots$$

$$+ C_\ell \overset{\ell-1}{\underset{n=1}{\pi}} P_{a_n n} \overset{a_\ell}{\underset{j=0}{\Sigma}} (1 - P_{j\ell})] - M [C_1 P_o \overset{a_1}{\underset{j=0}{\Sigma}} (1 - P_{j1}) + \ldots$$

$$+ C_{h+1} \prod_{n=1}^{h-1} P_{a_n n} \ P_{a_{h+1} h+1} \sum_{j=0}^{a_{n+1}} (1 - P_{j,h+1}) + C_h \prod_{n=1}^{h-1} P_{a_n n} \sum_{j=0}^{a_h} (1 - P_{jh}) +$$

$$\ldots + C_\ell \prod_{n=1}^{\ell-1} P_{a_n n} \sum_{j=0}^{a_\ell} (1 - P_j)] \le 0, \text{ where } P_0 = 1.$$

Performing the indicated operations and dividing both sides by the common terms M and $\prod_{n=1}^{h-1} P_{a_n n}$ yields:

$$[C_h \sum_{j=0}^{a_h} (1 - P_{j, h}) + C_{h+1} (P_{a_h h}) \sum_{j=0}^{a_{h+1}} (1 - P_{jh})] - \qquad (22)$$

$$[C_{h+1} \sum_{j=0}^{a_{h+1}} (1 - P_{j,h+1}) + C_h (P_{a_{h+1}, h+1}) \sum_{j=0}^{a_h} (1 - P_{jh})] \le 0,$$

which may be further reduced to

$$C_h (1 - P_{a_{h+1}, h+1}) \sum_{j=0}^{a_h} (1 - P_{jh}) \le C_{h+1} (1 - P_{a_h h}) \sum_{j=0}^{a_{h+1}} (1 - P_{j, h+1}). \quad (23)$$

Dividing both sides of equation (10) by $(1 - P_{a_{h+1}h+1})(1 - P_{a_h h})$ gives

$$\frac{C_h \sum\limits_{j=0}^{a_h} (1 - P_{jh})}{(1 - P_{a_h h})} \leq \frac{C_{h+1} \sum\limits_{j=0}^{a_{h+1}} (1 - P_{j,h+1})}{(1 - P_{a_{h+1},h+1})} \quad \text{or,} \quad (24)$$

$$\frac{(1 - P_{a_h h})}{C_h \sum\limits_{j=0}^{a_h} (1 - P_{jh})} \leq \frac{(1 - P_{a_{h+1},h+1})}{C_{h+1} \sum\limits_{j=0}^{a_{h+1}} (1 - P_{j,h+1})} \quad (25)$$

Constraining Output

Lemma 4.

(i) Given an optimally ordered event sequence in all aptitude tracks and $1 < EI_1 \leq EI_2 \ldots \leq EI_s$, where EI_i = average cost per graduate of the ith aptitude group, the greatest increase in efficiency, Max ΔEI, may be achieved by reducing the number of men output in the sth aptitude track.

(ii) Given an optimally ordered event sequence in all aptitude tracks and $EI_1 \leq EI_2 \leq EI_3 \ldots \leq EI_s < 1$, where EI_i = average cost per graduate of the ith aptitude group, the least decrease in efficiency, Min ΔEI_i, may be achieved by reducing the number of men output in the sth aptitude track.

(iii) Given an optimally ordered event sequence in all aptitude tracks and $EI_1 = EI_2 = \ldots = EI_s = 1$, where EI_i = average cost per

graduate of the ith aptitude group, no change in EI is possible by decreasing or increasing input.

Proof.

(i) Using equation (7) and substituting EI_i for C_{ik} and M_i^* for M_i in equation (16) yields

$$EI' = \frac{\sum\limits_{i=1}^{s} EI_i M_i^* - E_s \; \Delta M_s}{\sum\limits_{i=1}^{s} M_i^* - \Delta M_s} < EI \text{ where} \tag{26}$$

$$EI' = \frac{EI_1 M_1^* + \ldots + EI_s M_s^* - EI_s \; \Delta M_s}{\sum\limits_{i=1}^{s} M_i^* - \Delta M_s}, \tag{27}$$

$$\text{and } EI = \frac{EI_1 M_1^* + \ldots + EI_s M_s^* - EI_s \; \Delta M_s}{\sum\limits_{i=1}^{s} M_i^* - \Delta M_{s-1}} \text{ by Lemma 1 (ii).} \tag{28}$$

(ii) Using equation (7) and substituting EI_i for C_{ik} and M_i^* for M_i in equation (15) yields

$$EI' = \frac{\sum\limits_{i=1}^{s} EI_i M_i^* - E_s \; \Delta M_s}{\sum\limits_{i=1}^{s} M_i^* - \Delta M_s} > EI \text{ where} \tag{26}$$

$$EI' = \frac{EI_1 M_1^* + \ldots + EI_s M_s^* - EI_s \Delta M_s}{\sum\limits_{i=1}^{s} M_i^* - \Delta M_s} \quad , \qquad (27)$$

and $EI = \dfrac{EI_1 M_1^* + \ldots + EI_s M_s^* - EI_{s-1} \Delta M_{s-1}}{M_i^* - \Delta M_{s-1}}$. Thus $\Delta M_{s-1} = \Delta M_s$, (28)

by Lemma 1 (i).

(iii) Using equation (7) and substituting EI_i for C_{ik} and M_i^* for M_i in equation (17) yields $EI' = 1$ and $EI = 1$ for any ΔM_r.

It follows by the same reasoning that if ΔM_o cannot be achieved by continued input reductions in the highest cost track, continued reductions in input must be made in the next highest cost track. Output may be reduced in the sth aptitude track by reducing input into the sth track or, reducing the probability of success in the sth aptitude track by reducing the number of training trials.

The Reduction of Input Method

Input into the sth track should be reduced such that $M_s \prod\limits_{k=1}^{n} P_{ia_{ik}k} = M_s^*$ is satisfied (from equation (5)). A one unit reduction in input will result in a change of $\prod\limits_{i=1}^{n} P_{ia_{ik}k}$ units of output with a total cost of

$$C_s = (M_s - 1) \sum\limits_{k=1}^{n} C_{rk} \prod\limits_{\ell=0}^{k-1} P_{ra_{r\ell}\ell} \sum\limits_{j=0}^{a_{rk}} (1 - P_{ijk})$$

(from equation (6)).

The Reduction of the Probability of Success in the Sth Aptitude Track

The probabilities of success in one or all events may be regulated by varying the number of trials completed. As the number of trials decreases the probability of success decreases in any event. The probabilities of success should be regulated such that equation (5) is satisfied as nearly as possible. A weakness exists in this procedure. It is a result of the fact the P_{ik} are not continuous functions of the number of trials completed. That is the P_{ik} are not continuous over the range (0, 1). Consequently, the ideal adjustment of P_{Sk} necessary to satisfy $M_S^* = M_S P_{S1} \cdot P_{S2} \cdot \ldots \cdot P_{Sn}$ precisely may not be possible.

A reduction in the number of trials in a given event will:

(1) Reduce the cost of training within that event.

(2) Reduce the output from that event which reduces the input to the next event and, consequently, the cost of training in all succeeding events.

(3) Change the value of the ordering ratio, equation (14), such that the events must be reordered to retain an optimal sequence.

The following method of reducing trials to constrain output produces the least cost outcome.

(1) Determine the maximum system output (M_o) when unconstrained (equation (3)).

(2) If $M_o > M_o^*$, the desired output, then go to step 4.

(3) If $M_o < M_o^*$, the quantity or the quality of the input must be upgraded.

(4) Select all the $P_{S \cdot k}$ combinations in the Sth aptitude level which satisfy $M_S^* = M_o^* - [\sum\limits_{i=1}^{s-1} M_i \prod\limits_{k=1}^{n} P_{ik}]$, (from equations (3) and (4)).

(5) If $M_o < \sum\limits_{i=1}^{s-1} M_i \prod\limits_{k=1}^{n} P_{ik}$, then make further output reductions by this procedure in the S-1$^{\text{st}}$ track.

(6) Reorder the event sequence for each $(P_{s\cdot k})$ combination by the optimal sequence rule (equation (14)). The optimally ordered $(P_{s\cdot k})$ combinations constitute the constrained output strategies.

(7) Calculate the expected cost, expected output and EI (equations (2), (3), and (6)) for each strategy.

(8) Select the most efficient strategy.

Cost Constraints

A procedure similar to that used to constrain output by the trial reduction method may be used to constrain training costs. This procedure is:

(1) Determine the cost of the maximal, optimally order system output (C*).

(2) Let C_o = desired training costs. If $C_o < C*$ then select those P_{ik} combinations in the most costly aptitude track (e.g. i = 2) which satisfy

$$C_s = C_o - \sum_{i=1}^{s} \sum_{k=1}^{n} M_i C_{ik} \prod_{\ell=0}^{k-1} P_{ia_{i\ell}}^{a_{ik}} \sum_{j=0} (1 - P_{ijk})$$

where

$$C_s^* = M_s \sum_{k=1}^{n} C_{sk} \prod_{=0}^{k-1} P_{sa_{i\ell}}^{a_{sk}} \sum_{j=0} (1 - P_{sjk})$$

subject to j, an integer, $M_i \geq 0$ and $0 \leq P_{ik} \leq 1$.

(3) Proceed as before in steps 3, 4 and 5 of the reduced input procedure.

Summary

The development of the cost criterion functions and the expected output equations provide the measurement devices necessary to compare the two training procedures by the system performance criterion of instructor hours per graduate. The lemmas establish the relationships between input quality and quantity and cost per unit of output. This information together with the optimal order sequence rule for the proposed procedure provide some guarantee that should the EI's for the two procedures be compared, it is possible to compare best to best. Such a comparison and applications of Lemmas 1, 2, 3 and 4 and Theorem 1 are the subject of the next chapter.

CHAPTER V

APPLICATIONS TO A SIMULATED TRAINING SYSTEM

In this chapter, applications of the criterion functions, the lemmas, and the theorem developed in Chapter IV are made to a simulated training system. The sensitivity of the efficiency index (EI) to changes in training procedure, ordering of the training events, input quality, and output quantity are examined.

It was found that the EI of the simulated training system is sensitive to changes in training procedure. Specifically, the present training procedure uses more instructor hours per graduate than does the proposed procedure, given the estimated cost coefficients developed in Appendix E. It was also found that the simulated system EI is sensitive to changes in certain operating conditions when training is conducted by either the present or proposed procedures. A reduction in the quality of input reduces efficiency, but an increase in the quality of input increases the system's efficiency when total output remains constant (Lemmas 3 (i) and 4 (i)). The test system EI is also sensitive to efforts to constrain output. If output is constrained by reducing input to the lowest aptitude (highest cost) track, EI decreases (improves) under the proposed procedure (Lemma 4 (i)) but under the present procedure the EI increases (worsens; Lemma 1 (iii)). Constraining input is the only output control available under the present procedure. Two methods of constraining output, given the proposed procedure, are demonstrated. The first, Option 1, reduces output

by reducing input. The second, Option 2, reduces output by reducing

the probability of success for one or several events. The probability

of success in a given event is reduced by reducing the number of trials

in that event. Both options result in a decrease in EI when the output

reductions are made in the highest cost tracks (Lemma 4 (i)).

The simulated system EI is sensitive to changes in the event

order when operating under the proposed procedure. Arranging the events

by use of the optimal event sequence rule (Theorem 1) produces the best

EI for both constrained and unconstrained output conditions.

In the following sections, empirical evidence is presented in

support of the conclusions just summarized.

EI of the Test System - Present Procedure Versus Proposed Procedure for Average, High, and Low Quality Inputs

An average quality input (40 high aptitude, 120 average aptitude,

and 40 low aptitude men) was used initially for both procedures. The

cost coefficients were estimated as explained in Appendix E. The calcu-

lation of the cost of operating the test system by the present procedure

was accomplished first; equation (1) was used. The cost was found to

be 2753.8 instructor hours. Next the expected output for the present

procedure was calculated using the equation $M_o = M_I P$.

An explanation of the value of P used in the calculation of ex-

pected output is relevant at this point. It has been historically

established (8, 14) that 98 percent of the trainees entering basic

training successfully graduate. If the empirical evidence presented

by Fox, _et al._, is used to calculate P, P = .79 for an average quality

input, P = .84 for a high quality input, and P = .57 for a low quality

input. P = .98 is perhaps the result of some synergetic effect in the training system. It is conjectured that were the proposed procedure introduced into the real training environment that a similar synergism would take place.

It is not possible to calculate the EI for both procedures using both the historical P = .98 and the empirical P = .79. EI is a function of cost and probability of failure for both procedures. The cost calculation for the proposed procedure is a function of the learning rates, the number of trials, the trial costs, and the expected output from the preceding event. Learning rates for the test system events which would correspond to a terminal probability of success of .98 are not known. Therefore, it is impossible to calculate the training costs which are necessary to calculate an EI for the test system operated by the proposed procedure. The calculation of an EI under the present procedure using both the historical and the empirical probabilities of success is possible, however. The results of a sample calculation are given below.

The expected output of the present procedure was calculated using the historical P = .98 and the empirical P = .79 for average quality input. The respective expected outputs were 196 graduates and 158.38 graduates with corresponding EI's of 13.8 instructor hours per graduate and 17.3 instructor hours per graduate. The EI for the proposed procedure was 5.7 instructor hours per graduate as calculated by the computer program presented in Appendix B.

High Quality Input

In a like manner, the present and proposed procedures were

compared when a high quality input (120 high, 40 average, and 40 low aptitude men) was introduced to the simulated system. The results were that the present procedure generated EI's of 12.21 and 15.1 instructor hours per graduate versus an EI of 4.27 instructor hours per graduate for the proposed procedure.

Low Quality Input

Both procedures were also used to "train" a low quality input. The low quality input consisted of 40 high, 40 average, and 120 low aptitude men. The EI's were 18.86 and 32.6 instructor hours per graduate for the present procedure versus 10.22 for the proposed procedure.

It was concluded from these experiments that the proposed procedure is more efficient than the present procedure given high, normal, and low quality inputs. It was also concluded that decreases in input quality decrease the test system's efficiency regardless of the procedure used.

Table 1. EI for the Present and the Proposed Procedures
Under a Variety of Input Conditions

Input Aptitude	Present Procedure				Proposed Procedure		
	Grads	Cost	EI_1	EI_1'	Grads	Cost	EI_2
High	196	2393.6	12.2	15.1	167.98	717.38	4.27
Average	196	2753.8	13.8	17.3	158.38	903.57	5.70
Low	196	3696.5	18.9	32.6	113.55	1160.0	10.22

EI of the Test System Operated by Both Procedures When Output Is Constrained by Constraining Input

As mentioned in the preceding section, the probability of success in the BCT system under the present procedure, regardless of input quality, is .98. The factors that cause $P = .98$ have not been identified. Consequently, it is not known how to regulate these factors which affect the probability of success. Since the value of P could not be regulated to reduce the expected output from the simulated system operated by the present procedure, input was reduced. As input in the low quality group was reduced, the EI of the simulated system showed decreases in efficiency under the present procedure. As input was reduced in the low quality group under the proposed procedure, efficiency increased. It was found that when input in the low quality group was reduced the present procedure was less efficient than the proposed procedure.

The conditions established for comparing the present and proposed procedure included:

(1) The proposed procedure events would be optimally ordered. (Theorem 1)

(2) One hundred fifty trainee graduates were desired.

(3) Output would be constrained by reductions in input in the most efficient manner. (Lemmas 1 (i) and 4 (i)).

(4) The input available consists of 200 men; 40 high aptitude, 120 average aptitude, and 40 low aptitude.

A relevant question for both procedures is, "How much must input be reduced in the low quality track to achieve the desired output?" For the present procedure the answer is straightforward. Suppose M_o, the expected maximum possible output, is greater than M_o^*, the desired output.

It is known that $PM_I = M_O$ and that P is a constant (P = .98). It is also known that $M_I = M_1 + M_2 + M_3$. Therefore, M_3 input must be reduced such that the following constraint is satisfied: $P(M_1 + M_2 + M_3^*) = M_O^*$ (Lemma 2 (i)).

The procedure is not too dissimilar for the proposed procedure once the trial levels have been set. Using equation (6) and letting $\sum_{k=1}^{n} P_{ijk} = P_i$, i=1,2,3, $M_O = M_1P_1 + M_2P_2 + M_3P_3$. Suppose M_O, the expected maximum output, is greater than M_O^*, the desired output. Then the constraint $M_3^* = M_3P_3$ must be satisfied where M_3^* is determined by equation (8): $M_3^* = M_O^* - M_1P_1 + M_2P_2$.

The following empirical results were obtained:

Table 2. Constrained Inputs

	Present Procedure (P = .98)	Proposed Procedure
Input	154	174
M_1	40	40
M_2	114	120
M_3	0	14
Desired Output	150	150
Actual Output	150.92	150.1
M_1	39.2	40
M_2	111.72	105.6
M_3	0	4.5
Total Cost	1716.57	704.86
EI	15.057	4.70

When the above results were compared to the unconstrained input results, it was observed that the proposed procedure becomes more efficient as low aptitude input is reduced (Lemma 4 (i)). The present system becomes less efficient under the same conditions (Lemma 1 (i)). The proposed system is more efficient (3.2 times as efficient) than the present system when input is constrained under the stated conditions.

EI of the Test System Operated by the Proposed Procedure When Output is Constrained by Reducing Training Trials

Suppose the option to reduce input is not allowed while the system is being operated by the proposed procedure. If reductions in input cannot be made and output must be constrained, reductions in the number of trials in any or all aptitude groups reduces the cumulative probability of success. The reduction of the probability of success in turn reduces the expected output of the system. To do this in a systematic manner, the procedure presented in Chapter IV, page 46, was used.

A sample results summary is presented from the outcomes for a strategy from each ability group. The events by title and number in the following tables are

Rifle Assembly - 1

Rifle Disassembly - 2

Missile Preparation - 3

Military Symbols - 4

Phonetic Alphabet - 5

Combat Plotting - 6

Table 3.   Reductions in High Quality Group Output

Aptitude Level	Events (k) (in order)	Trials Completed	P_{ik}	Output	Cumulative Cost	EI
	2	8	.9			
	5	3	1.0			
	4	3	1.0			
1	1	9	1.0			
	6	3	1.0			
	3	7	1.0	36	73.17	2.0325
	1	13	.88			
	2	15	1.0			
	5	4	1.0			
2	4	7	1.0			
	6	7	1.0			
	3	8	1.0	105.6	594.22	4.94
	6	9	.76			
	4	11	.78			
	1	13	.80			
3	2	15	.90			
	5	7	.96			
	3	14	.78	12.78	896.15	23.63
Total				154.38	896.15	5.80

Table 4. Reductions in Average Quality Group Output

Aptitude Level (i)	Events (k) (in order)	Trials Completed	P_{ik}	Output	Cumulative Cost	EI
1	5	3	1.0			
	2	12	1.0			
	4	3	1.0			
	1	9	1.0			
	6	3	1.0			
	3	7	1.0	40	80.59	2.015
2	1	13	.88			
	3	7	.94			
	2	15	1.0			
	5	4	1.0			
	4	7	1.0			
	6	7	1.0	99.26	591.98	5.16
3	6	9	.76			
	4	11	.78			
	1	13	.80			
	2	15	.90			
	5	7	.96			
	3	14	.78	12.78	893.91	23.63
Total				152.05	893.91	5.88

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Table 5. Reductions in Low Quality Group Output

Aptitude Level	Events (k) (in order)	Trials Completed	P_{ik}	Output	Cumulative Cost	EI
	5	3	1.0			
	2	12	1.0			
	4	3	1.0			
1	1	9	1.0			
	6	3	1.0			
	3	7	1.0	40	80.59	2.015
	1	13	.88			
	2	15	1.0			
	5	4	1.0			
2	4	7	1.0			
	6	7	1.0			
	3	8	1.0	105.6	601.64	4.94
	6	9	.76			
	4	11	.78			
	1	13	.80			
3	2	15	.90			
	3	7	.28			
	5	7	.96	4.37	705.76	23.83
Total				149.97	705.76	4.71

The results of the preceding exercises indicate that the most efficient attempt to constrain output was made by reducing the number of trials and consequently the output in the low aptitude group (Lemma 4). All possible P_{3k} combinations for reductions in the low aptitude track (strategies) were used to calculate the EI's for the test system. For the particular data, the strategy which called for a reduction from 14 to 7 trials in event 5 with a corresponding change in P_{35} from .96 to .28 proved to be the most efficient. The final result of this strategy, shown in Table 5, was an EI of 4.71 instructor hours per graduate cost with 149.97 expected graduates.

The trial reduction method of constraining output does have some weaknesses, but it does provide the BCT manager with a mechanism for controlling output in addition to reducing input. This method is not as efficient as the input reduction method in the examples just presented, however. The reason is that the trial reduction method trains the unreduced input for a few trials before they are failed out of the training system. Comparing the best results by the trial reduction method and the best results previously presented for the input reduction method illustrates this point. For example:

	Method	
	Input Reduction	Trial Reduction
Input	174	200.00
Graduates	150.1	149.97
EI	4.7 $\frac{\text{inst. hrs.}}{\text{grad.}}$	4.71 $\frac{\text{inst. hrs.}}{\text{grad.}}$

The difference of .01 instructor hours per graduate is attributable to the expense of training the 26 additional men in the input stream for

those few trials before they failed out of the training system. This conclusion cannot be generalized. It is a particular outcome for the data used.

EI of Test System Operated by the Proposed Procedure When Event Orders Are Changed

The efficiency of the test system was greatly influenced by the order of the training events in the test system. The observation of this fact is what originally provided the motivation for the development of an optimal sequence rule. The outcomes of three nonoptimally and one optimally arranged (by Theorem 1) event sequences are summarized below. For this analysis an average quality input of 200 men (40 high aptitude, 120 average aptitude, and 40 low aptitude men) was used; outputs were not constrained.

Applications of the optimal sequence rule were also made to constrained output cases and variable input quality cases. Those results are not summarized in this section. They have already been presented in this chapter in Tables 1 through 5.

Summary

The findings from the preceding applications lend empirical support to the more general conclusions of Chapter IV. Particularly, applications of Lemmas 1, 2, 3, and 4 and Theorem 1 were demonstrated. Although no general lemma or theorem was developed which concluded that the present procedure is less efficient than the proposed procedure, the empirical findings demonstrate that is true for the particular simulated training system used to compare them.

Table 6. Various Event Orderings

Cost (Inst. Hrs.)	Graduates	EI (Inst. Hrs. Per Grad)	Apt Trk: Event Order
1065.19	158.38	6.73	(Nonoptimal) 1: 1, 2, 3, 4, 5, 6 2: 1, 2, 3, 4, 5, 6 3: 1, 2, 3, 4, 5, 6
1207.92	158.38	7.63	(Nonoptimal) 1: 1, 2, 3, 4, 5, 6 2: 1, 2, 3, 4, 5, 6 3: 3, 1, 2, 4, 6, 5
1251.67	158.38	7.92	(Nonoptimal) 1: 1, 2, 3, 4, 5, 6 2: 3, 1, 2, 4, 5, 6 3: 3, 1, 2, 4, 5, 6
903.57	158.38	5.70	(Optimal) 1: Any order 2: 1, 2, 3, 4, 5, 6 3: 6, 4, 1, 2, 5, 3

In addition to lending empirical support to the referenced lemmas and theorem, these applications were presented so that the reader might get some notion of the uses that management could make of the information provided by the criterion functions and constraint equations developed for both training procedures. The uses demonstrated are not all inclusive. Such an application as comparing the EI's of two different training units, given like inputs, thereby giving a relative measure of the effectiveness of the trainers or their methods of training might be desirable. There are perhaps others also.

It is not intended that the instructor hours per trainee graduate calculated for the test system in any way reflect the number of instructor hours per graduate now spent in BCT. The cost coefficients developed in Appendix E are merely estimates of the actual instructor hours that would be spent in similar types of subjects in BCT for each different aptitude group. As a consequence, the direct comparison of the simulated training system with actual BCT is not possible.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This study has developed a quantitative performance criterion for a manpower training system. Rules for minimizing the costs of training men in the simulated training system under a variety of constraint conditions have been developed, proven, and demonstrated.

The assumptions under which the proposed procedure and its underlying model were developed are:

(1) Men can be categorized into aptitude groups by the Armed Forces Qualification Test. Once categorized they remain in a group.

(2) Slower, less-apt trainees require more instructors and more time to learn a subject than do higher aptitude men.

(3) There is no probabilistic dependency between events. The probability of success for an event is independent of the event order in the event sequence.

(4) Men are not recycled between events.

(5) As quickly as a man successfully completes a trial test in any event, he proceeds to the next event in the sequence.

(6) A trial is a uniform time period of instruction-testing for all aptitude groups. Instructors are of equal capability and use identical teaching methods within an event.

(7) Men do not repeat trials in an event indefinitely. A maximum permissible number of trials is established for each event. Beyond this maximum no increase in the cumulative probability of success occurs.

(8) Instructor costs accrue only when men are training. Idle instructors have zero cost.

There are three general results of this thesis. They are presented in the order of their development.

The development of a quantifiable performance criterion for a training system generated the criterion function for the simulated system.

The development of Theorem 1, which used the criterion function for the proposed procedure, represents an extension of the work done by Mitten. The concept of optimally ordering stochastic events when the cost of an (inspection) event sequence is dependent upon cost and the probability of failure for each event was first presented by Mitten around 1960 (17). In the case presented by Mitten, the probability of failure and the cost for each event were known and these two values were independent. In the case presented by the author, the probability of failure for a given event and the cost of the event are both functions of the number of trials (attempts to succeed) within an event. As the number of trials increases, the cumulative probability of failure decreases and the event cost increases. Independent multiple event tracks are also dealt with by the author.

The development of the most efficient manner by which aptitude track output reductions can be made if reduced output is desired from the simulated training system (Lemma 4) provides a general solution to the constrained output problem for the proposed procedure.

The results of this thesis applicable to Basic Combat Training in the United States Army and research on it are now presented.

The proposed procedure could represent a 58 percent savings in BCT instructor costs. The proposed procedure must be field tested

before the true savings or losses can be determined accurately, however.

The development of the generalized relationships (Lemmas 1, 2, and 3) between input quality and quantity and the cost of operation of the simulated training system provide insight to the expected behavior of BCT were it operated by the proposed procedure.

The methodology for using the relationship between aptitude and performance presented by Fox, et al., permitted the development of a performance criterion and criterion functions for the test system. The developed criterion functions allow the comparison of the present and other training procedures for the simulated training system. The criterion measures may be compared under a variety of input and output conditions.

Recommendations

The application of the criterion functions to the simulated training system should be extended. The analytical extension of this study might examine the reaction of the system to continuous or phased inputs so that an optimal input flow or cycle may be determined for various input qualities. Decision criteria such as minimum instructor idle time and minimum trainee time in the system are appropriate for this analysis. The analytical extension of this study should also include the extension of Theorem 1. Theorem 1 treats only the case where the events in the system are independent. Some preliminary work has been done by Conway, et al., (10) which may serve as a guide to the extension of Theorem 1 to include a rule for optimally sequencing the events when inter-event dependencies exist.

Consideration should be given to other ways of improving the system's efficiency. What are the effects of morale, attitude, modes of training, and incentive situations on the rates of learning for the various aptitude groups? Further, by what means might output be increased beyond the maximum expected output? Must individual performance criteria levels be lowered or will better training methods yield the desired output?

Applications of the model of the proposed procedure in addition to those presented here might include predicting physical and marksmanship performance. A comparison of the effects of various training modes and attitude conditions may also be possible. The potential for application of the proposed procedure is not necessarily limited to BCT. Any sequential training system might be made more efficient by adopting a similar procedure.

It is recommended that the U.S. Army field test the proposed procedure to evaluate its actual efficiency and to develop more realistic cost coefficients. If the proposed procedure withstands application in field tests, the potential 58 percent savings in training costs per man would be significant in these times of limited budget.

..., is greater than M_o^*, the desired output.

APPENDIX A

LEARNING CURVES AND PROMPT FREQUENCIES
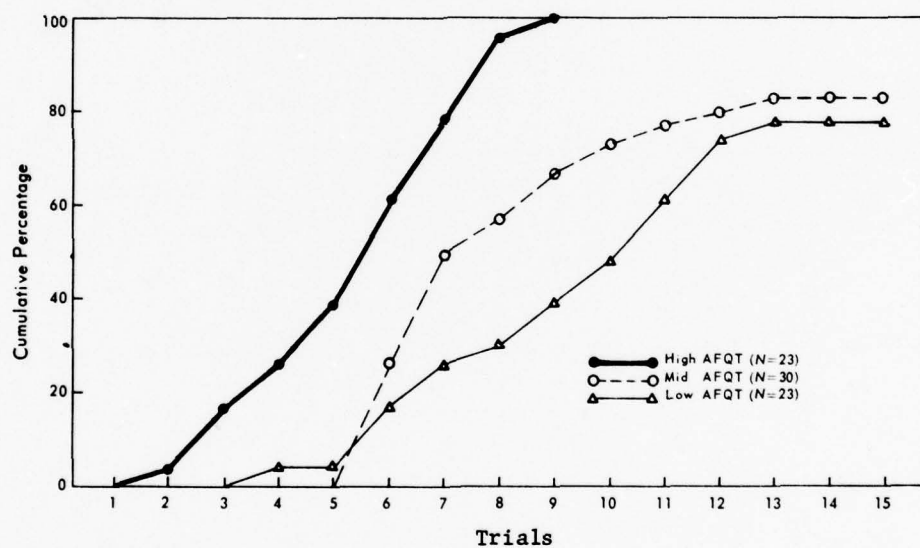
Figure 3. Rifle Assembly: Cumulative Percentage
of Subjects Reaching Criterion Per Trial (13).
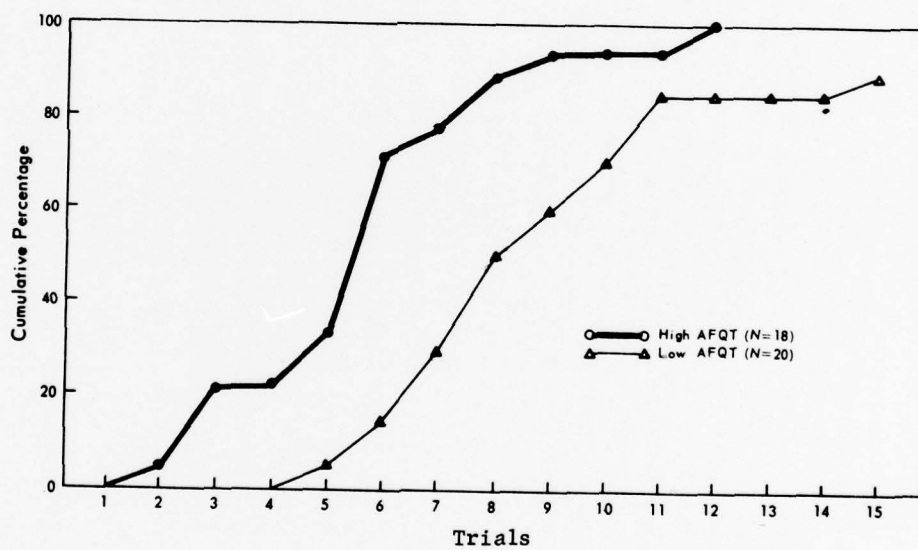
Figure 4. Rifle Disassembly: Cumulative Percentage
of Subjects Reaching Criterion Per Trial (13).
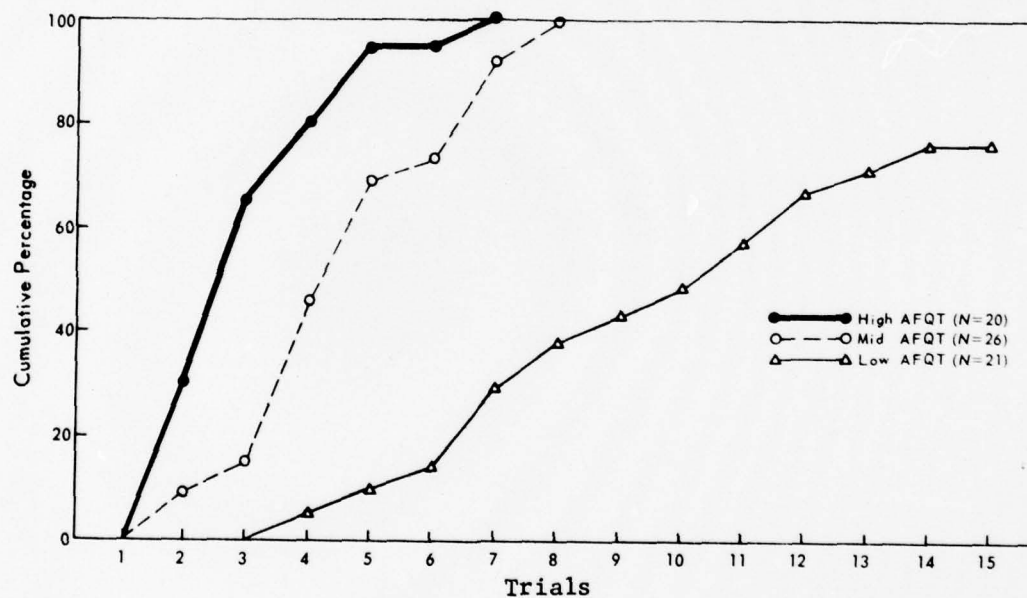
Figure 5.   Missile Task: Cumulative Percentage of
Subjects Reaching Criterion Per Trial (13).
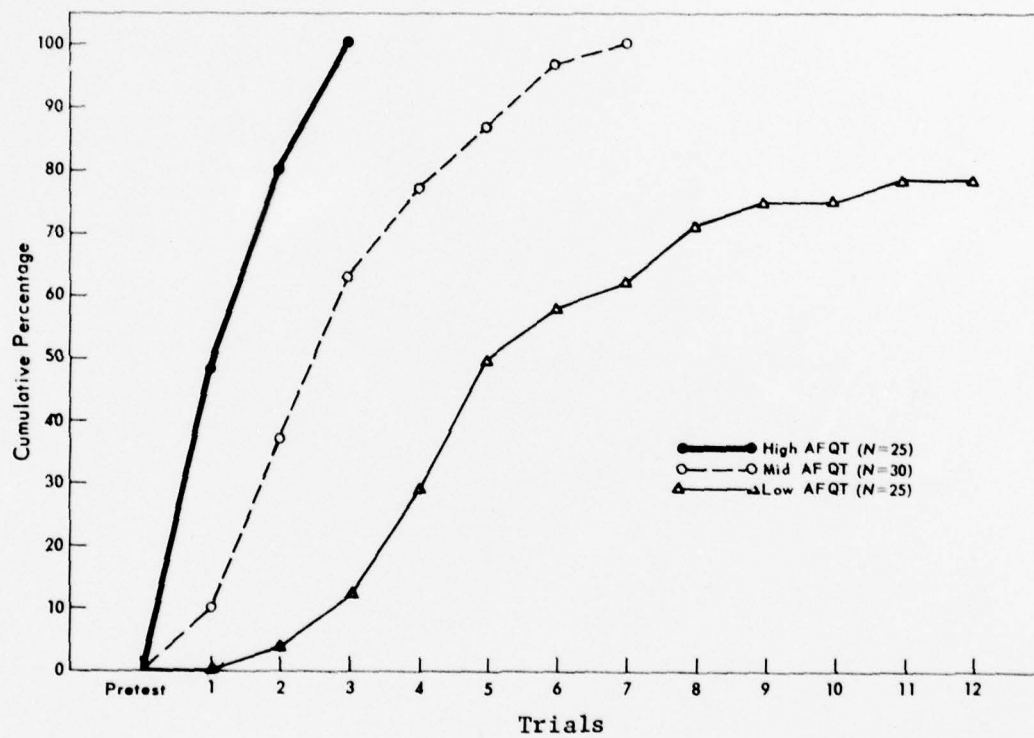
Figure 6. Military Symbols: Cumulative Percentage
of Subjects Reaching Criterion Per Trial (13).
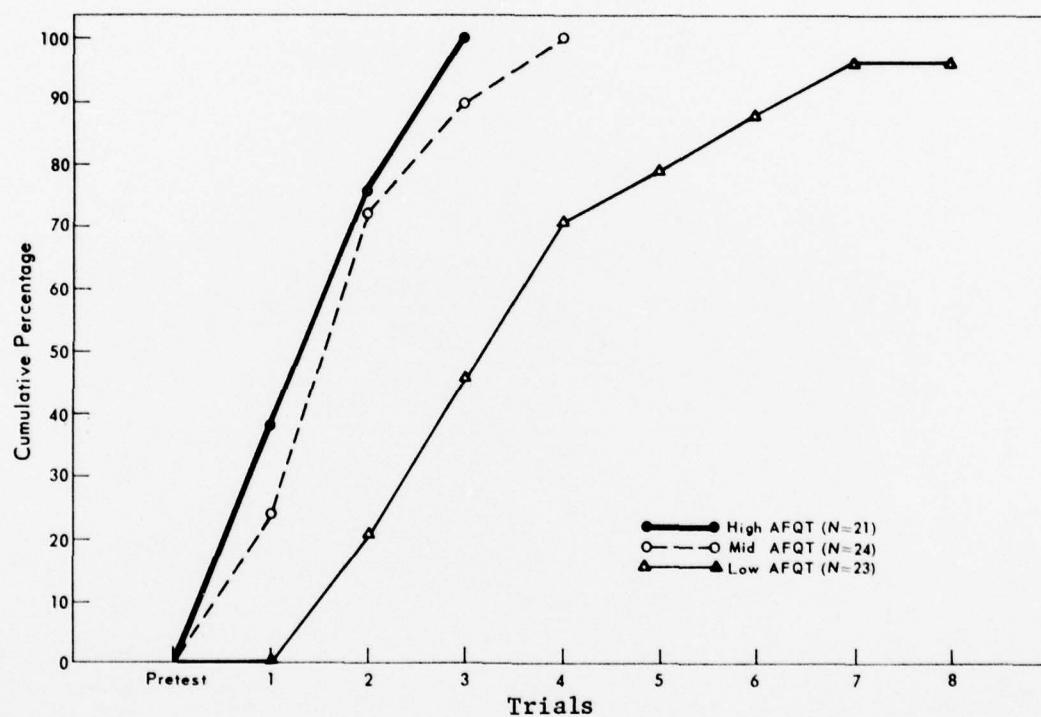
Figure 7.   Phonetic Equivalents: Cumulative Percentage
            of Subjects Reaching Criterion Per Trial (13).
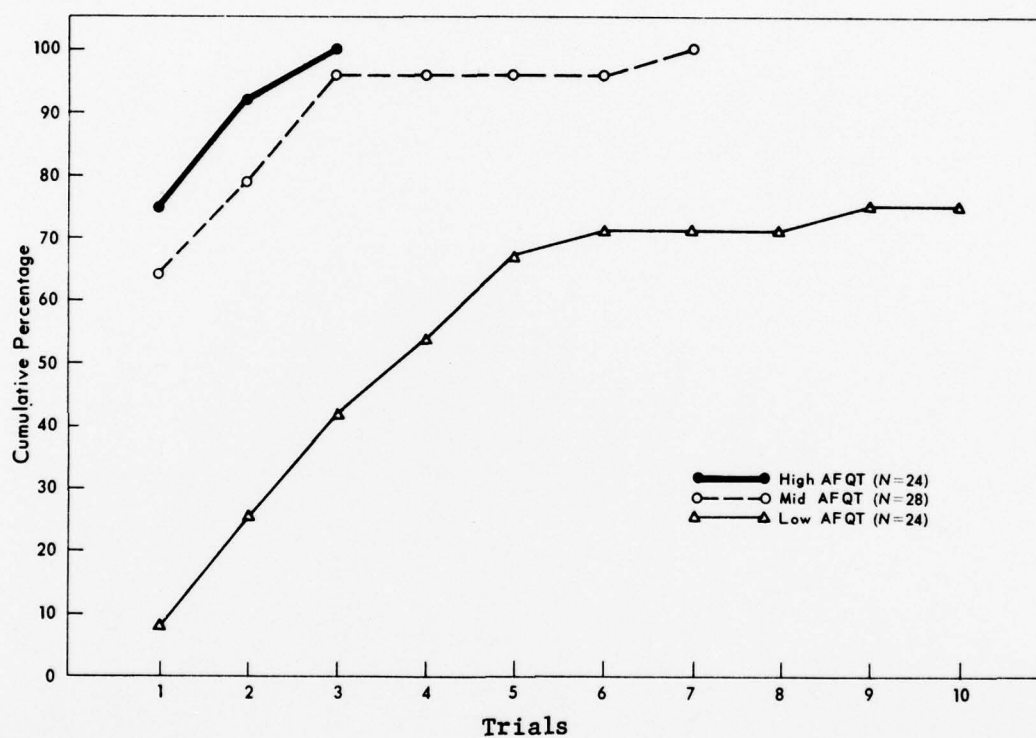
Figure 8. Combat Plotting: Cumulative Percentage
of Subjects Reaching Criterion Per Trial (13).

Table 7. Mean Prompt Frequencies (13)

Event	Aptitude Group		
	1	2	3
1. Rifle Assembly	6.4	11.3	16.6
2. Rifle Disassembly	5.4	9.2	12.2
3. Missile Task	22.9	42.9	133.0
4. Military Symbols*	1.7	3.3	6.2
5. Phonetic Equivalents*	1.9	2.1	4.0
6. Combat Plotting*	1.3	1.7	5.2

*Based on one prompt per trial.

APPENDIX B

COMPUTER PROGRAM FOR THE CALCULATION

OF TEST SYSTEM EFFICIENCY

```
00100      1*   C   RLDUCTION OF MAX FLOW TOWARD A GIVEN TOTAL OUTPUT
00101      2*          DIMENSION M(3),C(3,6,18),P(3,15,6,18),EVAL(3,6,18)
00103      3*          INTEGER TM(3,6,18),R
00104      4*          REAL MVAL,EFIND,P
00105      5*          DATA M(1),M(2),M(3)/40,40,120/
00111      6*          DO 100 R=1,18
00114      7*          DO 100 K=1,6
00117      8*          DO 100 I=1,3
00122      9*          READ(5,1)(P(I,J,K,R),J=1,15)
00130     10*        1 FORMAT(15(F4.2))
00131     11*      100 CONTINUE
00135     12*          DO 101 R=1,18
00140     13*          DO 101 I=1,3
00143     14*          READ(5,2)(C(I,K,R),K=1,6)
00151     15*        2 FORMAT(6(F6.4,2X))
00152     16*      101 CONTINUE
00155     17*          DO 56 R=1,18
00160     18*       56 READ(5,3)((TM(I,K,R),I=1,3),K=1,6)
00172     19*        3 FORMAT(I2,17I3)
00173     20*          WRITE(6,21)
00175     21*       21 FORMAT(21X,'EXPECTED  EVENT    CUMULATIVE  CUMULATIVE')
00176     22*          WRITE(6,20)
00200     23*       20 FORMAT(12X,'EVENT    OUTPUT    COST     OUTPUT      COST')
00201     24*        7 DO 202 R=1,18
00204     25*          TCST=0
00205     26*          MVAL=0
00206     27*          DO 201 I=1,3
00211     28*          DUMMY=M(I)
00212     29*          SVAL=0
00213     30*          DO 200 K=1,6
00216     31*          TO=0
00217     32*          ECST=0
00220     33*          NJ=TM(I,K,R)
00221     34*          DO 199 J=1,NJ
00224     35*          CST=C(I,K,R)*(DUMMY-TO)
00225     36*          TO=DUMMY*P(I,J,K,R)
00226     37*          ECST=ECST+CST
00227     38*          WRITE(6,15) ECST,J,TO
00234     39*       15 FORMAT(33X,F6.2,62X,'TRIAL=',I2,2X,'TO=',F6.2)
00235     40*      199 CONTINUE
00237     41*          J=TM(I,K,R)
00240     42*          EVAL(I,K,R)=DUMMY*P(I,J,K,R)
00241     43*          DUMMY=EVAL(I,K,R)
00242     44*          TCST=TCST+ECST
00243     45*          WRITE(6,11) I,K,EVAL(I,K,R),CST,TCST,J
00253     46*       11 FORMAT(4X,'I=',I2,5X,'K=',I2,2X,F8.2,3X,F8.2,14X,F8.2)
00254     47*      200 CONTINUE
00256     48*          SVAL=EVAL(I,6,R)
00257     49*          WRITE(6,14) SVAL
00262     50*       14 FORMAT(40X,F8.2)
00263     51*          MVAL=MVAL+SVAL
00264     52*      201 CONTINUE
00266     53*          EFIND=TCST/MVAL
00267     54*          WRITE(6,13) M(1),M(2),M(3), MVAL,EFIND,R
00277     55*       13 FORMAT(/,5X,'MAX OUTPUT,GIVEN M(1)= ',I3,1X,'M(2)= ',I3,1X,'M(3)= '
00277     56*         1',I3,1X,'IS= ',F8.2,5X,'EFFICIENCY INDEX= ',F8.2,'INST HRS/TRME',/
00277     57*         1/,50X,'R= ',I2)
00300     58*      202 CONTINUE
00302     59*          END
```

END OF COMPILATION: NO DIAGNOSTICS.

```
I= 3   K= 4   51.22        56.0?                    TRIAL=11  TO= 47.80
                           57.11                    TRIAL=12  TO= 47.80
                           58.1?                    TRIAL=13  TO= 47.80
                           59.1?                    TRIAL=14  TO= 47.80
                           60.1?        522.44      TRIAL=15  TO= 51.22
                            1.0?

I= 3   K= 5   49.17         3.71                    TRIAL= 1  TO=   .00
                            5.43                    TRIAL= 2  TO= 11.27
                            7.55                    TRIAL= 3  TO= 23.56
                            ?.?1                    TRIAL= 4  TO= 36.88
                            ?.77                    TRIAL= 5  TO= 40.97
                           1?.3?                    TRIAL= 6  TO= 45.07
                           1?.64        533.00      TRIAL= 7  TO= 49.17
                             .3?

                           61.95                    TRIAL= 1  TO=   .00
                          123.91                    TRIAL= 2  TO=   .00
                          185.06                    TRIAL= 3  TO=   .00
                          247.81                    TRIAL= 4  TO=  1.97
                          307.29                    TRIAL= 5  TO=  3.93
                          364.2?                    TRIAL= 6  TO=  7.87
                          416.3?                    TRIAL= 7  TO= 13.77
                          468.9?                    TRIAL= 8  TO= 19.67
                          49?.1?                    TRIAL= 9  TO= 21.63
                          53?.??                    TRIAL=10  TO= 23.60
                          565.??                    TRIAL=11  TO= 29.50
                          599.7?                    TRIAL=12  TO= 33.44
                          609.62                    TRIAL=13  TO= 35.40
I= 3   K= 6   ?0.35       62?.97       1160.04      TRIAL=14  TO= 38.35
                           17.35

                               38.35

MAX OUTPUT GIVEN M(1)= 40  M(2)= 40  M(3)= 120 IS=  113.55     EFFICIENCY INDEX=   10.22TNST HRS/TRNE

                                  D= 1
```

APPENDIX C

COMPUTER PROGRAM FOR THE CALCULATION OF THE

OPTIMAL SEQUENCE ORDERING RATIOS

```
00100   1*   C  THIS PROGRAM CALCULATES THE RATIO USED TO SEQUENCE THE EVENTS IN THE SYSTEM
00101   2*         DIMENSION P(3,15,6),C(3,6)
00103   3*         REAL OR,SUMP
00104   4*         DO 10 K=1,6
00107   5*         DO 10 I=1,3
00112   6*         READ(5,90)(P(I,J,K),J=1,15)
00120   7*      90 FORMAT(15(F4.2))
00121   8*      10 CONTINUE
00124   9*         DO 20 I=1,3
00127  10*         READ(5,95)(C(I,K),K=1,6)
00135  11*      95 FORMAT(6(F6.4,2X))
00136  12*      20 CONTINUE
00140  13*         WRITE(6,100)
00142  14*     100 FORMAT(5X,'APT LEVEL',2X,'EVENT',2X,'NO OF TRIALS',2X,'SUM OF PROB
00142  15*        1',2X,'ORDERING RATIO')
00143  16*         DO 25 I=1,3
00146  17*         DO 25 K=1,6
00151  18*         DO 25 NJ=1,15
00154  19*         SUMP=0
00155  20*         DO 30 J=1,NJ
00160  21*         SUMP=SUMP+P(I,J,K)
00161  22*      30 CONTINUE
00163  23*         OR=(1-F(I,NJ,K))/C(I,K)*(NJ-SUMP)
00164  24*         WRITE(6,101) I,K,NJ,SUMP,OR
00173  25*     101 FORMAT(9X,I2,6X,I2,8X,I2,11X,F5.2,9X,F7.3)
00174  26*      25 CONTINUE
00200  27*         END
```

```
@XQT
MAP U017-05/18-17:40
START=010233, PROG SIZE(I/D)=3837/2398
```

APT LEVEL	EVENT	NO OF TRIALS	SUM OF PROB	ORDERING RATIO
1	1	1	.40	7.200
1	1	2	1.18	3.608
1	1	3	2.18	.000
1	1	4	3.18	.000
1	1	5	4.18	.000
1	1	6	5.18	.000
1	1	7	6.18	.000
1	1	8	7.18	.000
1	1	9	8.18	.000
1	1	10	9.18	.000
1	1	11	10.18	.000
1	1	12	11.18	.000
1	1	13	12.18	.000
1	1	14	13.18	.000
1	1	15	14.18	.000
1	2	1	.48	10.816
1	2	2	1.28	5.760
1	2	3	2.28	.000
1	2	4	3.28	.000
1	2	5	4.28	.000
1	2	6	5.28	.000
1	2	7	6.28	.000
1	2	8	7.28	.000
1	2	9	8.28	.000
1	2	10	9.28	.000
1	2	11	10.28	.000
1	2	12	11.28	.000
1	2	13	12.28	.000
1	2	14	13.28	.000
1	2	15	14.28	.000
1	3	1	.00	40.000
1	3	2	.04	75.264
1	3	3	.20	94.080
1	3	4	.44	108.224
1	3	5	.84	99.840
1	3	6	1.48	65.098
1	3	7	2.28	37.760
1	3	8	3.24	7.616
1	3	9	4.24	.000
1	3	10	5.24	.000
1	3	11	6.24	.000
1	3	12	7.24	.000
1	3	13	8.24	.000
1	3	14	9.24	.000
1	3	15	10.24	.000
1	4	1	.74	1.352
1	4	2	1.66	.544
1	4	3	2.66	.000
1	4	4	3.66	.000
1	4	5	4.66	.000
1	4	6	5.66	.000
1	4	7	6.66	.000
1	4	8	7.66	.000

1	4	9	8.66	.000
1	4	10	9.66	.000
1	4	11	10.66	.000
1	4	12	11.66	.000
1	4	13	12.66	.000
1	4	14	13.66	.000
1	4	15	14.66	.000
1	5	1	.00	50.000
1	5	2	.32	57.120
1	5	3	1.00	32.000
1	5	4	1.80	22.000
1	5	5	2.76	4.480
1	5	6	3.72	4.560
1	5	7	4.72	.000
1	5	8	5.72	.000
1	5	9	6.72	.000
1	5	10	7.72	.000
1	5	11	8.72	.000
1	5	12	9.72	.000
1	5	13	10.72	.000
1	5	14	11.72	.000
1	5	15	12.72	.000
1	6	1	.00	2.500
1	6	2	.32	2.856
1	6	3	1.00	1.600
1	6	4	1.80	1.100
1	6	5	2.76	.224
1	6	6	3.72	.228
1	6	7	4.72	.000
1	6	8	5.72	.000
1	6	9	6.72	.000
1	6	10	7.72	.000
1	6	11	8.72	.000
1	6	12	9.72	.000
1	6	13	10.72	.000
1	6	14	11.72	.000
1	6	15	12.72	.000
2	1	1	.00	10.000
2	1	2	.00	20.000
2	1	3	.04	28.416
2	1	4	.14	34.740
2	1	5	.32	38.376
2	1	6	.77	28.765
2	1	7	1.31	26.174
2	1	8	2.01	17.970
2	1	9	2.78	14.306
2	1	10	3.59	12.179
2	1	11	4.48	7.172
2	1	12	5.43	3.285
2	1	13	6.38	3.310
2	1	14	7.33	3.335
2	1	15	8.33	.000
2	2	1	.24	6.795
2	2	2	.98	3.120
2	2	3	1.88	1.318
2	2	4	2.88	.000
2	2	5	3.88	.000

2	2	6	4.88	.000
2	2	7	5.88	.000
2	2	8	6.88	.000
2	2	9	7.88	.000
2	2	10	8.88	.000
2	2	11	9.88	.000
2	2	12	10.88	.000
2	2	13	11.88	.000
2	2	14	12.88	.000
2	2	15	13.88	.000
2	3	1	.10	28.929
2	3	2	.48	33.657
2	3	3	1.10	25.786
2	3	4	1.88	16.657
2	3	5	2.76	9.600
2	3	6	3.72	3.257
2	3	7	4.72	.000
2	3	8	5.72	.000
2	3	9	6.72	.000
2	3	10	7.72	.000
2	3	11	8.72	.000
2	3	12	9.72	.000
2	3	13	10.72	.000
2	3	14	11.72	.000
2	3	15	12.72	.000
2	4	1	.64	2.672
2	4	2	1.42	2.631
2	4	3	2.38	.511
2	4	4	3.34	.544
2	4	5	4.30	.577
2	4	6	5.30	.000
2	4	7	6.30	.000
2	4	8	7.30	.000
2	4	9	8.30	.000
2	4	10	9.30	.000
2	4	11	10.30	.000
2	4	12	11.30	.000
2	4	13	12.30	.000
2	4	14	13.30	.000
2	4	15	14.30	.000
2	5	1	.00	38.462
2	5	2	.08	67.938
2	5	3	.24	89.169
2	5	4	.68	71.508
2	5	5	1.36	44.800
2	5	6	2.08	42.215
2	5	7	3.02	9.185
2	5	8	4.02	.000
2	5	9	5.02	.000
2	5	10	6.02	.000
2	5	11	7.02	.000
2	5	12	8.02	.000
2	5	13	9.02	.000
2	5	14	10.02	.000
2	5	15	11.02	.000
2	6	1	.00	1.639
2	6	2	.08	2.896

2	6	3	.24	3.801
2	6	4	.68	3.048
2	6	5	1.36	1.910
2	6	6	2.08	1.799
2	6	7	3.02	.391
2	6	8	4.02	.000
2	6	9	5.02	.000
2	6	10	6.02	.000
2	6	11	7.02	.000
2	6	12	8.02	.000
2	6	13	9.02	.000
2	6	14	10.02	.000
2	6	15	11.02	.000
3	1	1	.00	12.500
3	1	2	.04	23.520
3	1	3	.16	31.240
3	1	4	.44	32.040
3	1	5	.94	25.375
3	1	6	1.54	22.300
3	1	7	2.18	21.690
3	1	8	2.90	17.850
3	1	9	3.66	16.020
3	1	10	4.42	16.740
3	1	11	5.20	15.950
3	1	12	5.98	16.555
3	1	13	6.76	17.160
3	1	14	7.54	17.765
3	1	15	8.32	18.370
3	2	1	.00	10.989
3	2	2	.00	21.978
3	2	3	.00	32.967
3	2	4	.04	41.776
3	2	5	.08	51.903
3	2	6	.24	53.169
3	2	7	.48	54.453
3	2	8	.76	57.284
3	2	9	1.16	51.692
3	2	10	1.64	47.771
3	2	11	2.24	38.505
3	2	12	3.00	23.736
3	2	13	3.80	20.220
3	2	14	4.60	20.659
3	2	15	5.40	21.099
3	3	1	.00	6.667
3	3	2	.00	13.333
3	3	3	.00	20.000
3	3	4	.00	26.667
3	3	5	.04	31.744
3	3	6	.20	32.480
3	3	7	.50	30.333
3	3	8	1.00	23.333
3	3	9	1.60	19.733
3	3	10	2.28	16.469
3	3	11	3.12	8.405
3	3	12	3.96	8.576
3	3	13	4.80	8.747
3	3	14	5.64	8.917
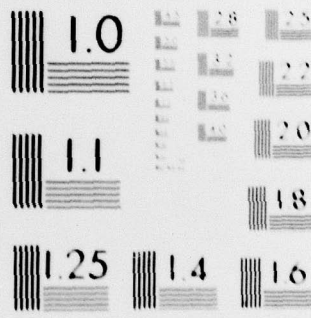
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

3	3	15	6.54	5.640
3	4	1	.00	8.850
3	4	2	.22	12.287
3	4	3	.68	11.087
3	4	4	1.40	6.442
3	4	5	2.20	4.956
3	4	6	3.08	3.101
3	4	7	4.04	1.048
3	4	8	5.00	1.062
3	4	9	5.96	1.076
3	4	10	6.92	1.090
3	4	11	7.88	1.104
3	4	12	8.84	1.119
3	4	13	9.80	1.133
3	4	14	10.76	1.147
3	4	15	11.72	1.161
3	5	1	.00	18.868
3	5	2	.00	37.736
3	5	3	.00	56.604
3	5	4	.04	71.728
3	5	5	.12	84.709
3	5	6	.28	90.657
3	5	7	.56	87.487
3	5	8	.96	79.698
3	5	9	1.40	80.302
3	5	10	1.88	79.668
3	5	11	2.48	64.302
3	5	12	3.16	53.374
3	5	13	3.88	48.181
3	5	14	4.66	38.770
3	5	15	5.44	39.683
3	6	1	.00	.794
3	6	2	.00	1.587
3	6	3	.00	2.381
3	6	4	.04	3.017
3	6	5	.12	3.563
3	6	6	.28	3.813
3	6	7	.56	3.680
3	6	8	.96	3.352
3	6	9	1.40	3.378
3	6	10	1.88	3.351
3	6	11	2.48	2.705
3	6	12	3.16	2.245
3	6	13	3.88	2.027
3	6	14	4.66	1.631
3	6	15	5.44	1.669

APPENDIX D

Table 8. Cumulative Probabilities

Event (k)	Trials (j)	Cumulative Probability By Aptitude Group (i)		
		P_{1jk}	P_{2jk}	P_{3jk}
1	1	0	0	0
	2	.04	0	0
	3	.16	0	0
	4	.24	0	.04
	5	.40	0	.04
	6	.64	.28	.16
	7	.80	.48	.24
	8	.96	.56	.28
	9	1.0	.68	.40
	10	1.0	.72	.48
	11	1.0	.80	.60
	12	1.0	.84	.76
	13	1.0	.88	.80
	14	1.0	.88	.80
	15	1.0	.88	.80
2	1	0	0	0
	2	.04	0	0
	3	.20	.04	0
	4	.20	.10	0
	5	.32	.18	.04
	6	.74	.45	.16
	7	.78	.54	.30
	8	.90	.70	.50
	9	.94	.77	.60
	10	.94	.81	.68
	11	.94	.89	.84
	12	1.0	.95	.84
	13	1.0	.95	.84
	14	1.0	.95	.84
	15	1.0	1.0	.90

Table 8. Cumulative Probabilities
(Continued)

Event (k)	Trials (j)	Cumulative Probability By Aptitude Group (i)		
		P_{1jk}	P_{2jk}	P_{3jk}
3	1	0	0	0
	2	.32	.08	0
	3	.68	.16	0
	4	.80	.44	.04
	5	.96	.68	.08
	6	.96	.72	.16
	7	1.0	.94	.28
	8	1.0	1.0	.40
	9	1.0	1.0	.44
	10	1.0	1.0	.48
	11	1.0	1.0	.60
	12	1.0	1.0	.68
	13	1.0	1.0	.72
	14	1.0	1.0	.78
	15	1.0	1.0	.78
4	1	.48	.10	0
	2	.80	.38	.04
	3	1.0	.62	.12
	4	1.0	.78	.28
	5	1.0	.88	.50
	6	1.0	.96	.60
	7	1.0	1.0	.64
	8	1.0	1.0	.72
	9	1.0	1.0	.76
	10	1.0	1.0	.76
	11	1.0	1.0	.78
	12	1.0	1.0	.78
	13	1.0	1.0	.78
	14	1.0	1.0	.78
	15	1.0	1.0	.78

Table 8. Cumulative Probabilities
(Continued)

Event (k)	Trials (j)	Cumulative Probability By Aptitude Group (i)		
		P_{1jk}	P_{2jk}	P_{3jk}
5	1	.40	.24	0
	2	.78	.74	.22
	3	1.0	.90	.46
	4	1.0	1.0	.72
	5	1.0	1.0	.80
	6	1.0	1.0	.88
	7	1.0	1.0	.96
	8	1.0	1.0	.96
	9	1.0	1.0	.96
	10	1.0	1.0	.96
6	1	.74	.64	.08
	2	.92	.78	.26
	3	1.0	.96	.42
	4	1.0	.96	.54
	5	1.0	.96	.68
	6	1.0	1.0	.72
	7	1.0	1.0	.72
	8	1.0	1.0	.72
	9	1.0	1.0	.76
	10	1.0	1.0	.76

(Note: Table 8 Data was developed from Figures 3 through 8, Annex A.)

APPENDIX E

CALCULATION OF COST COEFFICIENTS

Cost Coefficients for the Proposed Procedure

The cost coefficients are estimates of the number of instructor hours per student required to conduct a trial in each event for each of three aptitude groups. It was left to the author's experience to estimate the number of hours and the number of instructors required to conduct a trial in each event. (Both of these estimates were made based on the description of the nature of each event by Fox, et al., (13) and eight years military experience in training men and being trained in skills similar to those found in the six test events.) The combination of these two estimates provided an estimated, normative number of instructor hours per trial for each event regardless of aptitude track.

In the test system Fox, et al., kept a record of the number of prompts by ability group required to assist a trainee in the performance test for each event. A prompt was defined to be any assistance offered by an instructor to a trainee during a trainee performance test (13). The prompt data were used to establish a relative frequency of need for instructor assistance by the trainees. This plus the normative estimate of the number of instructor hours per trial for each event were used to calculate the cost coefficients for each aptitude track in each event. The frequency of prompts for the high ability group,

f_1, was used as the norm frequency of prompts in each event. All frequencies were compared to f_1 for each event as follows:

$$\frac{f_1}{f_1} = r_1 = 1, \text{ for all events.}$$

$$\frac{f_2}{f_1} = r_2 = \frac{9.2}{5.4} = 1.7, \text{ for } k = 2 \text{ as an example.}$$

$$\frac{f_3}{f_1} = r_3 = \frac{12.2}{5.4} = 2.26, \text{ for } k = 2 \text{ as an example.}$$

The relative frequency multiplier (r_i) was then multiplied by the estimated number of hours per trial, k_{ik}, and the estimated instructors per 100 students, t_k, to establish the cost in instructor hours per trainee trial for each ability group in each event. That is $C_{ik} = r_i k_{ik} t_k$. Dimensionally,

$$C_{ik} = r_i \frac{\text{(hours)}}{\text{(trial)}} \cdot \frac{\text{(instructors)}}{\text{(trainees (100))}} = \frac{\text{instructor hours}}{\text{trainee trial}} .$$

For example: $C_{22} = (1.7)(1 \text{ hour/trial}) \frac{(5 \text{ instructors})}{(100 \text{ trainees})} = .085$ inst hrs/ trial.

Cost Coefficients For the Present Procedure

The cost of training one man to graduation is equal to the sum of the costs of training him in each event. The cost of training him in any event is equal to the number of units of training time spent in each event times the cost of each unit. Using the cost coefficients developed for the proposed procedure in conjunction with variable quality inputs, M_i, the cost per man per training time unit for each

event was calculated. $C_k = \dfrac{\displaystyle\sum_{i=1}^{3} C_{ik}M_i}{\displaystyle\sum_{i=1}^{3} M_i}$. For example:

$$C_3 = \frac{.4\ \frac{\text{inst hr}}{\text{trial}}\ (40\ \text{men}) + .61\ \frac{\text{inst hr}}{\text{trial}}\ (120\ \text{men}) + 1.26\ \frac{\text{inst hr}}{\text{trial}}\ (40\ \text{men})}{40\ \text{men} + 120\ \text{men} + 40\ \text{men}} =$$

$.698\ \dfrac{\text{inst hrs}}{\text{man trial}}$. The C_k developed by this procedure were then used directly in equation (1) to calculate total training costs. Cost coefficients for the present and proposed procedures are presented in tabular form in Tables 9 through 11.

Table 9. Instructor Hours/Trainee Trial for the Proposed Procedure

For Event (k)	Cost Coefficients By Aptitude (i) Per Trainee Trial		
	1	2	3
1	.050	.100	.150
2	.050	.085	.113
3	.400	.610	1.26
4	.025	.0485	.091
5	.025	.028	.053
6	.020	.026	.080

Table 10.   Instructor Hours/Trainee Trial for the Present Procedure
With Average Quality Input

For Event (k)	Cost Coefficient per Trainee Trial (C_k)
1	.10
2	.0836
3	.698
4	.0523
5	.0324
6	.0360

Table 11.   Instructor Hours/Trainee Trial for the Present Procedure
With High Quality Input

For Event (k)	Cost Coefficient per Trainee Trial (C_k)
1	.0800
2	.0696
3	.6140
4	.0429
5	.0312
6	.0332

Table 12. Instructor Hours/Trainee Trial for the Present Procedure
With Low Quality Input

For Event (k)	Cost Coefficients per Trainee Trial (C_k)
1	.12
2	.0948
3	.958
4	.0693
5	.0424
6	.0572

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